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Optimisation techniques of urban water distribution networks

Tehnici de optimizare a rețelelor urbane de distribuție a apei

Prof.emeritus dr.eng. Ioan SARBU[1], Assoc.prof.dr.eng. Adriana TOKAR[1], Drd.eng. Simona POPA-ALBU[1]

[1]*Polytechnic University of Timisoara*
Piața Victoriei 2, Timișoara, Romania
email: ovidiu.sirbu@umft.ro, adriana.tokar@upt.ro,

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Abstract: *The distribution network is an essential part of all urban water supply systems that requires efficient design and operation, which may be achieved through effective application of optimisation methods. This paper provides a briefly overview of the most approached methods for optimisation of water distribution networks (WDNs) design and operation. The main deterministic and heuristic optimisation techniques are synthesised and described, and several optimisation models in literature, which used these methods for WDN design/rehabilitation and operation are indicated. Finally, the advantages and disadvantages of the optimisation techniques for urban WDNs are presented.*

Keywords: *water distribution, pipe network, optimal design, deterministic methods, heuristic techniques.*

Rezumat: *Rețeaua de distribuție este o parte esențială a tuturor sistemelor urbane de alimen-tare cu apă care necesită o proiectare și funcționare eficientă, se se poate realiza prin aplicarea eficace a metodelor de optimizare. Această lucrare oferă o scurtă prezentare a celor mai abordate metode pentru optimizarea proiectării și funcționării rețelelor de distribuție a apei (RDA). Principalele tehnici de optimizare deterministe și heuristice sunt sintetizate și descrise și sunt indicate mai multe modele de optimizare din literatura de specialitate, care au utilizat aceste metode pentru proiectarea/reabilitarea și funcționarea sistemelor urbane de distribuție a apei. În final, sunt prezentate avantajele și dezavan-tajele tehnicilor de optimizare a RDA urbane.*

Cuvinte cheie: *distribuția apei, rețea de conducte, dimensionare optimală, metode deterministe, tehnici heuristice.*

1. Introduction

Distribution system costs within any water supply scheme may be equal to or greater than 60% of the entire cost of the project [1,2]. These observations highlight the need for an efficient and safe water distribution network (WDN). The reduction of the cost and energy consumption of the WDN can be achieved through its design and operational optimisation. An important stage of network design is to find the optimum network layout which satisfies requirements such as pressure, power consumption and demands at different nodes and also to minimise cost while meeting a performance criterion. The development of WDNs without the use of optimisation provides non-optimal structures, based essentially on the immediate response to the growing water demand of population and industry [1]. These non-optimal structures are translated into non-efficient systems in terms of design and operation. The unpredictability of growing water demand also creates a challenge for optimisation techniques. For these reasons, recourse to the optimisation tools is crucial. For the optimal design of WDNs both steady and transient states must be taken into consideration.

Optimisation problems can be solved using conventional trial and error methods or more effective optimisation methods. However, in WDNs, the optimisation process by trial and error methods can present difficulties due to the complexity of these systems such as multiple pumps, valves and reservoirs, head losses, large variations in pressure values, several demand loads, etc. For this reason, innovative linear [3], non-linear [4, 5] and heuristic [6-11] optimisation algorithms are becoming more widely explored in optimisation processes of the WDNs. In the solution procedure, each algorithm is linked with a hydraulic analysis solver of WDNs to obtain the optimum solution. Consideration of reliability in WDNs also has been drawing increasing attention over the past few years [12, 13].

This paper provides a briefly overview of the most approached methods for optimisation of water distribution networks (WDNs) design and operation. The main deterministic and heuristic optimisation techniques are synthesised and described, and several optimisation models in literature, which used these methods for WDN design/rehabilitation and operation are indicated. Finally, the advantages and disadvantages of the optimisation techniques for urban WDNs are presented.

2. Methods and techniques of optimisation

Due to the complexities in the optimal design of WDNs, many researchers have applied diverse suitable calculation methods to solve the problem. The optimisation methods and techniques can be classified into two main categories: (1) *deterministic* methods, based essentially on the computation of the objective function gradient and/or function evaluations, and (2) *heuristic* techniques, based essentially on exploratory search and natural phenomena or even on artificial intelligence. Heuristic searches that use the heuristic function in a strategic way are referred to as *meta- heuristic* techniques.

2.1 Deterministic methods

The deterministic methods most applied in WDN optimisation comprise linear programming (LP), integer linear programming (ILP), non-linear programming (NLP), integer non-linear programming (INLP), and dynamic programming (DP). Optimisation problems that combine continuous and integer values are referred to as *mixed-integer* programming (MIP). These kinds of algorithms enable finding the exact position of an optimal solution. However, they usually converge to local optimal solutions which may not be the global optimum. In addition, the need of derivative evaluations can, in some cases, complicate the optimisation process.

• *Linear programming* (LP) consists of determining the minimum (ma-ximum) of the linear objective function F with several unknown *decision variables* x_i linked by a system with a number of linear equations and inequations which represent the constraints:

subject to : $F = c_1 x_1 + c_2 x_2 + ... + c_n x_n \rightarrow \min(\max)$

$$\begin{aligned} & a_{11} x_1 + a_{12} x_2 + ... a_{1n} x_n \le b_1 \\ & a_{21} x_1 + a_{22} x_2 + ... a_{2n} x_n \le b_2 \\ & .... \\ & a_{m1} x_1 + a_{m2} x_2 + ... a_{mn} x_n \le b_m \\ & x_j \ge 0 \ ; \ j=1,2,....,n \end{aligned} \tag{1}$$

where c_j ($j = 1,\dots,n$) are the constant coefficients of the objective function; a_{ij} ($i = 1,\dots,m$; $j = 1,\dots,n$) are the constant coefficients of the constraints; and b_i ($i = 1,...,m$) are the free terms in the system of constraints.

The most commonly used algorithm for solving the LP model is the *Simplex algorithm* [14], which has been developed in several variants. Mays and Tung [15] recommended strongly the use of the LP method in designing pipe networks. LP works well for pipe sizing problems involving branched networks with one-directional flow. However, Sarbu and Ostafe [3] developed a mathematical model and a numerical procedure based on LP for optimal design of a looped pipe network supplied from one or more sources, according to demand variation operating in a transitory turbulence flow, considering the pipe lengths as decision variables.

• *Non-linear programming* (NLP) is a method of optimising problems that can be described by a non-linear objective function or/and some non-linear constraints. The model of NLP with constraints is defined by equations:

subject to

$$\begin{aligned} & F(X) \rightarrow \min(\max) \\ & \varphi_j(X) \ \{\ge; =; \le\} 0; \ j=1,2,....m \end{aligned} \tag{2}$$

Where, the objective function F and the constraints φ_j constitute continuous non-linear functions, and $\mathbf{X}=\{x_1, x_2,\dots,x_n\}$ is the vector of decision variables with dimension n.

NLP can be classified into *convex programming* and *non-convex program-ming*. The *conditional gradients* method can be used to minimise a non-linear function in a domain in which the minimum of a linear function is obtained without any difficulty and the well-known *Newton-Raphson* method is quite good when the trial solution is close to the optimum but can be quite unreliable for solutions far from the optimum [16].

Different NLP methods, such as the *substitution* and *Lagrange multipliers* methods, or a specific case of NLP such as *quadratic programming* (QP), were detailed by Hillier and Lieberman [17]. The generalised *reduced gradient* (RG) has been used for WDN design [18] and WDN operation [19]. Sarbu and Kalmar [5] developed an improved non-linear optimisation model for looped networks supplied by direct pumping based on NLP. This problem of NLP with equality constraints finally turns into a system of non- linear equations to be solved by the „gradient method".

• *Dynamic programming* (DP) is a method of optimising a multistage decision process in which a decision is required at each stage. This method is based on Bellman's optimality principle [20], which, in short, is formulated as follows: "an optimal policy can only be made up of optimal sub-policies." To solve a DP problem, it is necessary to evaluate both immediate and long-term consequence costs for each possible state at each stage. This evaluation is done through the development of the following recursive functional equation (for minimisation):

$$\min Z = f_{0,i}(X_0, X_i) = \min_{\{x_{i-1}\}}[V_i(X_{i-1}, X_i) + f_{0,i-1}(X_0, X_{i-1})] \quad (3)$$

Where, $V_i(X_{i-1}, X_i)$, (i = 1, 2... N) are the costs attached to each stage; Z is the cost function attached for the set of stages; N is the total number of stages; the terms X_i are vectors with n components $\{x_{1i}, x_{2i}, \ldots x_{ni}\}$; and the notation $\{X_{i-1}\}$ means that X_{i-1} belongs to a values set which depend only on X_0 and X_i.

Because DP works for a series of stages, the only types of design problems it is applicable to are single pipes with multiple withdrawals as described by Liang [21].

2.2 Heuristic techniques

The group of heuristic techniques mainly includes genetic algorithms (GAs), evolutionary algorithms (EAs), and other heuristic algorithms such as differential evolution (DE), cross- entropy (CE), shuffled frog leaping algorithm (SFLA), simulated annealing (SA), tabu search (TS) algorithm, particle swarm optimi- sation (PSO), ant-colony optimisation (ACO), harmony search (HS), etc. Heuristic searches that use the heuristic function in a strategic way are referred to as *meta-heuristic* techniques. These techniques provide the advantages of not requiring derivatives calculations and do not rely on the initial choice of values for the decision variables. Due to the exploratory nature of the heuristic algorithms, the probability of finding global optimal solutions using these advanced techniques is higher than in the case of deterministic methods. The main disadvantage of these techniques is related to the higher computational effort [22].

• *Genetic algorithm* (GA) is a powerful search technique based on the genetic process of biological organisms proposed by Holland [23]. The theory behind GAs was developed in the 1980s by Goldberg [24] and others. Murphy and Simpson [25] were the first to apply a GA on WDNs, followed by Simpson et al. [26].

The use of a GA involves the following five steps:

1. Randomly generate a set of individuals, which is called an *initial population*. Usually, a binary alphabet (characters may be 0 or 1) is used to form chromosomes represented as a binary string.

2. Compute the *fitness function* analogous to the *objective function,* which determines the ability of an individual to compete with other individuals in the initial population. A *penalty coefficient* incorporated in the objective function is activated for an infeasible solution (e.g., pressure violation).

3. Produce a new population using the *reproduction* (crossover) and *mutation* operators. The fittest individuals are selected for reproduction to produce offspring of the next generation.

4. Compute the fitness function of the new solutions.

5. Terminate the algorithm if the population has converged or repeat steps 3 through 5 to produce successive generations.

A GA based multi-objective optimisation tool called “GANetXL” for solving both single and multi-objective optimisation problems was initially developed by Savic et al. [27], and later used by many researchers in the field of water systems [28]. This optimisation tool is an add-on to Microsoft Excel, and it has the provision to link up with a hydraulic simulator such as EPANET [29] for constraint verification. From the family of multi- objective genetic algorithms, GANetXL incorporates the non- dominated sorting genetic algorithm II (NSGA-II) [30]. Creaco et al. [31] performed the optimal design of a new WDS considering two objectives, the construction costs and network reliability, using NSGA-II.

• *Differential evolution* (DE) algorithm was developed by Storn and Price [32] for optimisation problems over continuous domains. DE uses crossover and mutation operators to generate new solutions, but with two main differences compared to GA [8]. Accordingly, DE calculates the mutation size for a randomly selected solution a, from the population as described by Yazdi [33]: (1) Two members (say b and c) are selected randomly from the population;

(2) A multiplicative of “differential vector” is considered as mutation size: $\beta\times(b-c)$ where β is called scaling factor.

Then, a temporary solution is generated by the mutation as: $y=a+\beta\times(b-c)$. After doing mutation, crossover is carried out to generate a new solution $z=\{z_1, z_2,\ldots, z_n\}$ using the solution $x=\{x_1, x_2,\ldots, x_n\}$ of the population and temporary solution $y=\{y_1, y_2,\ldots, y_n\}$ obtained by mutation task.

Recently, Mansouri et al. [34] used DE to optimise the design of a branched WDN.

• *Cross-entropy* (CE) method is an adaptive algorithm based on variance minimisation [35]. CE method involves an iterative procedure in which iterations can be broken down into two phases: (1) generate a random data sample according to a specified mechanism; (2) update the parameters of the random mechanism based on the data to produce a "better" sample in the next iteration. Perelman and Ostfeld [36] applied this method to the optimisation of WDNs.

• *Shuffled frog leaping algorithm* (SFLA), introduced by Eusuff and Lansey [37], is a meta-heuristic technique whose operating principles are similar to other existing evolutionary techniques, which try to find an optimal solution to a problem from the evolution of an initial population. SFLA performs a heuristic search based on the evolution of particles called memes, carried by a number of interacting individuals (frogs) that perform a global exchange of information among the population. Mora-Melia et al. [38] presents a modified SFLA applied to the design of WDNs.

• *Simulated annealing* (SA) is a stochastic technique based on the physical annealing process in a solid material [39]. The SA method was adapted to be applied to the low cost design of WDNs [40]. Costa et al. [41] applied the SA technique for the optimal design of pipe networks including pumps.

• *Particle swarm optimisation* (PSO) is a concept developed by Kennedy and Eberhart [42], which has overcome the limitations of GA. Specifically, the PSO technique maintains a population of *particles*, each of which represents a potential solution to an optimisation problem. In this technique, the co-ordinates of each particle represent the possible solution and after each iteration, the particle moves towards optimal solution [43]. The convergence condition requires setting the move iteration number of the particle. Izquierdo et al. [44] have applied PSO in existing problems and concluded that PSO gives better results as compared to other classical methods like DP.

• *Ant-colony optimisation* (ACO) is a meta-heuristic algorithm based on the analogy of the foraging behaviour of a colony of ants, and their ability to determine the shortest route between their nest and an eating source by means of chemical pheromone (marker) trails [45]. In ACO algorithms, the optimisation search procedure is conducted by the number of artificial ants moving on a graph in the search space. Several special cases of the ACO meta-heuristic have been proposed in the literature such as the ant-system [46]. Ostfeld and Tubaltzev [47] linked an ant-colony scheme with EPANET for the minimisation of the system design and operation costs. Gil et al. [48] evaluated the performances of a new ACO implementation adapted to solve the single-objective constrained non-linear WDN for minimum investment. Much more interesting problem was approached by Abbasi et al. [49] like the design of a water adduction main under transient condition using ACO algorithm.

• *Harmony search* (HS) introduced by Geem and Kim [50] mimics the improvisation of music players. In its basic form, this technique starts by generating a set of random solutions called the harmony memory (HM), in which a predetermined number of harmonies have been stored, and then produces new solutions by sampling either from previously generated solutions in HM or from a random distribution. The best harmony stored in HM is returned as the found optimum solution. Geem and Cho [51] applied HS to the optimisation of WDNs. Baek et al. [52] employed HS to optimise the simulation of hydraulic under abnormal operating conditions in WDNs. The previously described existing meta- heuristic techniques can be divided into three classes [53]: (1) *local search meta-heuristics* (SA, TS) operate on a single complete solution and iteratively improve it by making small adjustments called moves; (2) *population-based meta-heuristics* (GA, DE, CE, SFLA) operate on a set of solutions and find better solutions by combining solutions from that set into new ones; and (3) *constructive meta-heuristics* (PSO, ACO, HS) build a solution by working with a single, unfinished, solution and adding one solution element at a time.

3. Conclusions

In this study, the general optimal WDN design problem was presented and various successful optimisation methods and techniques were reviewed.

The optimisation of pipe networks under steady-state conditions has been studied and different researchers proposed the use of mathematical programming techniques (LP, NLP, and DP) to identify the optimal solution for WDNs. However, these deterministic methods either use some gradient information or require restrictive assumptions such as linearity, convexity, and generally satisfied and they usually converge to local optimal solutions that may not be the global optimum.

Recently, the focus of the research in this area has shifted to the meta-heuristic based optimisation methods like GA, SA, ACO, PSO, SFLA, DE, HS, etc. As meta-heuristic optimisation methods use only the values of the objective function in the search for optimal solutions, a large number of numerical simu-lations are required to reach these solutions. This is time consuming for small problems, but for larger problems it may be the only feasible way, and in that sense the required computational effort is actually the benefit of this approach.

Further research in heuristic optimisation methods should focus on hybrid methods, which combine the specific advantages of different approaches. These studies should also contain the use of hyper-heuristic techniques for optimising WDNs, which are more general and can solve a wider series of problems compared to the current meta-heuristic methods specialised in a narrow class of problems.

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