

Steady state two-dimensional heat conduction in a square cross section, by using Microsoft Excel® for numerical modeling

Modelarea numerică a câmpului de temperatură 2D într-o secțiune pătrată, în regim termic staționar, utilizând aplicația software Microsoft Excel®

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Abstract. *The paper presents a simple way to study and numerically analyze the 2D temperature field in a square cross section of a beam. A new software tool was built and used for this study, using as the main calculation method the finite difference method (FDM). The steady state heat transfer through a body without internal heat sources was considered and the boundary conditions of the first kind (Dirichlet) were applied regarding the values of temperatures on the delimiting surfaces of the body. The solution of the linear equation system (LES) was obtained using the Solver tool from the Microsoft Excel® software application, finding the numerical image of the temperature distribution in the square cross section. Also, all the numerical results have been translated into a graphical form, for a more intuitive view.*

Keywords: heat, transfer, steady state, temperature, 2D, finite difference, Solver, Excel

Rezumat. *Lucrarea prezintă o modalitate de studiu și analiză numerică a câmpului de temperatură 2D într-o secțiune de formă pătrată a unei grinzi. A fost construit și utilizat pentru studiu un nou instrument software, folosind ca metodă de calcul principală metoda calculului cu diferențe finite (MDF). A fost considerat transferul termic în regim staționar, printr-un corp fără surse interioare de căldură și aplicate condițiile la limită de specia I (Dirichlet) privind valorile temperaturilor pe suprafețele delimitatoare ale corpului. Rezolvarea sistemului de ecuații liniare (SEL) format a fost efectuată cu ajutorul instrumentului Solver din aplicația software Microsoft Excel®, obținând determinarea numerică a distribuției temperaturii în câmpul secțiunii pătrate. Rezultatele numerice au fost transpuse și într-o formă grafică pentru o vizualizare mai intuitivă.*

Cuvinte cheie: transfer, căldură, staționar, câmp, distribuție, temperatură, 2D, diferențe, finite, sistem, ecuații, liniare, Solver, Excel

1. Introduction

Paper [1] has indicated the way by which, starting from the fundamental law of heat conduction (Fourier) in the general format (1) described by [2], [3], [4], [5], [6], [7], [8] etc.:

$$\frac{\partial t}{\partial \tau} = a \cdot \left(\nabla^2 t + \frac{\dot{q}_{SIC}}{\lambda} \right) \quad (1)$$

considering a steady state heat transfer $\left(\frac{\partial t}{\partial \tau} = 0 \right)$ and taking into account the fact that no internal heat sources (IHS) are present $(\dot{q}_{IHS} = 0)$, in the case when the temperature variation along one coordinate axis e.g., Oz may be neglected $\left(\frac{\partial t}{\partial z} = 0 \right)$, the Fourier equation is expressed in the simplified form (2) [2], [3], [5], [6] etc.

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} = 0 \quad (2)$$

The second-order partial differential equation (2) may be solved approximately by numerical integration techniques e.g., the finite differences method (FDM) [1], [2], [3], [4], [5], [6] etc., using equation (3) which is based on second-order finite differences

$$\frac{\Delta_x(\Delta t_x)}{\Delta x^2} + \frac{\Delta_y(\Delta t_y)}{\Delta y^2} = 0 \quad (3)$$

where $\Delta_x(\Delta t_x)$ and $\Delta_y(\Delta t_y)$ are expressed in the set of equations (4)

$$\begin{cases} \Delta_x(\Delta t_x) = (t_{x+\Delta x,y} - t_{x,y}) - (t_{x,y} - t_{x-\Delta x,y}) \\ \Delta_y(\Delta t_y) = (t_{x,y+\Delta y} - t_{x,y}) - (t_{x,y} - t_{x,y-\Delta y}) \end{cases} \quad (4)$$

and the first order finite differences, Δt_x and Δt_y are given, for example, in the set of equations (5)

$$\begin{cases} \Delta t_x = t_{x+\Delta x,y} - t_{x,y} \\ \Delta t_y = t_{x,y+\Delta y} - t_{x,y} \end{cases} \quad (5)$$

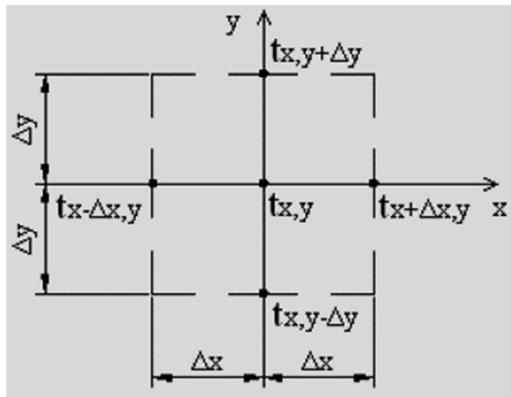


Fig. 1. The temperature $t_{x,y}$ in the node having coordinates (x,y) [1], [2], [3] etc.

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When a rectangular mesh (or grid) is considered for representing the nodes in which the temperatures will be calculated, as in Figure 1 [1], [3], [6], [7] etc., the generic temperature $t_{x,y}$ in the mesh node having coordinates (x,y) can be expressed by the means of equation (6):

$$t_{x,y} = \frac{\frac{t_{x+\Delta x,y} + t_{x-\Delta x,y}}{\Delta x^2} + \frac{t_{x,y+\Delta y} + t_{x,y-\Delta y}}{\Delta y^2}}{2 \cdot \left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} \right)} \quad (6)$$

In usual cases, a square mesh is used for the 2D analysis of the temperature field, by choosing the same spacing step $\Delta x = \Delta y = \Delta$ for nodal points generation on both coordinate axes Ox and Oy, so the generic temperature $t_{x,y}$ in a mesh node having coordinates (x,y) is calculated as an arithmetic average of the temperature at the four neighboring nodes (7) [1], [2], [3], [4], [5] etc.:

$$t_{x,y} = \frac{t_{x+\Delta,y} + t_{x-\Delta,y} + t_{x,y+\Delta} + t_{x,y-\Delta}}{4} \quad (7)$$

This paper presents a solution for the numerical calculation of the temperature distribution inside a square cross section, applying boundary conditions of the first kind (Dirichlet), by using the Solver instrument from Microsoft Excel®, with the purpose of solving the linear equation system (LES) obtained after running FDM.

The Solver instrument uses a group of cells from the Microsoft Excel® spreadsheet, directly or indirectly correlated with a certain cell called objective (Set Objective), which contains a value specified by the user (Value Of) or, in some other cases, may be maximized or minimized (Max/Min). With the goal to obtain in the objective cell the desired result by minimizing the computed numerical error, the Solver instrument modifies the initial values provided in certain dedicated cells (Variable Cells), takes into account the specified solving restrictions (Subject to the Constraints) that provide the frontiers of the solutions domain and finally calls a problem-solving algorithm, for example Simplex LP.

The proposed solving method is useful especially when initial data can suffer a series of modifications with respect to the situation studied in a certain solving stage, does not require a special effort from the user for the computer implementation and has the advantage of providing quick results, in both numerical and graphical form.

2. Solving the heat transfer problem

The technical application proposed for solving is described in the following statement:

A reinforced concrete (RC) beam with a square cross section of 40×40 cm, has on one of the surfaces the temperature $t_{s1}=150^{\circ}\text{C}$, and on the other surfaces the temperatures $t_{s2}=t_{s3}=t_{s4}=50^{\circ}\text{C}$ (Figure 2).

Using the finite differences method (FDM), determine the temperatures in the nodes of a squared discretization grid having an equal spacing step on the both Ox and Oy axes of coordinates, $\Delta x = \Delta y = 10$ cm.

Building the automatic calculation spreadsheet for this application requires the completion of certain work stages that will be described as follows.

First, it is necessary to specify and implement the initial data for the discussed problem i.e., the temperature values t_{s1} , t_{s2} , t_{s3} and t_{s4} on the delimiting surfaces of the square cross section (Figure 3).

It is preferable and recommended to specify the values of the temperatures by treating them as separate variables placed in independent cells, instead of using them as constant values placed directly into the formulas that will be inserted.

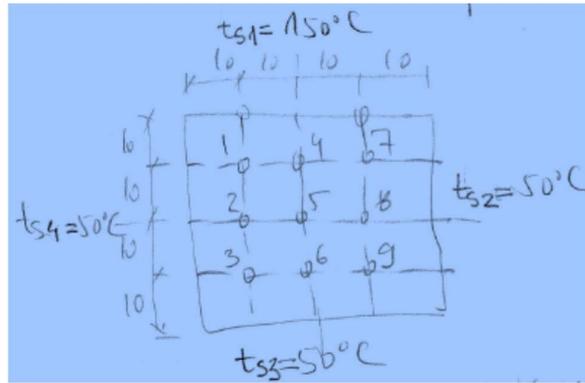


Fig. 2. Squared mesh for nodal temperature calculation (equal step $\Delta x = \Delta y = 10$ cm)

When these initial values may suffer some modifications, and especially when it is possible to have different combinations of those values for the study of the 2D field of temperature, the effort of adapting the calculation instrument to the new situation will be largely reduced, by having an easy access to the current values and quickly making the necessary changes.

Marime	ID	U.M.	Valoare
Temp. suprafata 1	ts1	150	C
Temp. suprafata 2	ts2	50	C
Temp. suprafata 3	ts3	50	C
Temp. suprafata 4	ts4	50	C

Fig. 3. Implementing the initial data of the problem (the temperatures t_{s1} , t_{s2} , t_{s3} , t_{s4})

Using relation (7) for expressing the unknown temperatures $t_1 \dots t_9$ in the nodes of the square discretization mesh from Figure 2, the next equations (8) were obtained

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$$\left\{ \begin{array}{l} t_1 = \frac{t_{s1} + t_4 + t_2 + t_{s4}}{4} \\ t_2 = \frac{t_1 + t_5 + t_3 + t_{s4}}{4} \\ t_3 = \frac{t_2 + t_6 + t_{s3} + t_{s4}}{4} \\ t_4 = \frac{t_{s1} + t_7 + t_5 + t_1}{4} \\ t_5 = \frac{t_4 + t_8 + t_6 + t_2}{4} \\ t_6 = \frac{t_5 + t_9 + t_{s3} + t_3}{4} \\ t_7 = \frac{t_{s1} + t_{s2} + t_8 + t_4}{4} \\ t_8 = \frac{t_7 + t_{s2} + t_9 + t_5}{4} \\ t_9 = \frac{t_8 + t_{s2} + t_{s3} + t_6}{4} \end{array} \right. \quad (8)$$

Using an intermediary step (9) in expressing the above equations

$$\left\{ \begin{array}{l} 4 \cdot t_1 = t_{s1} + t_4 + t_2 + t_{s4} \\ 4 \cdot t_2 = t_1 + t_5 + t_3 + t_{s4} \\ 4 \cdot t_3 = t_2 + t_6 + t_{s3} + t_{s4} \\ 4 \cdot t_4 = t_{s1} + t_7 + t_5 + t_1 \\ 4 \cdot t_5 = t_4 + t_8 + t_6 + t_2 \\ 4 \cdot t_6 = t_5 + t_9 + t_{s3} + t_3 \\ 4 \cdot t_7 = t_{s1} + t_{s2} + t_8 + t_4 \\ 4 \cdot t_8 = t_7 + t_{s2} + t_9 + t_5 \\ 4 \cdot t_9 = t_8 + t_{s2} + t_{s3} + t_6 \end{array} \right. \quad (9)$$

gives the LES (10), which has nine equations with nine unknowns and has to be solved numerically.

$$\left\{ \begin{array}{l} 4 \cdot t_1 - t_2 - 0 \cdot t_3 - t_4 + 0 \cdot t_5 + 0 \cdot t_6 + 0 \cdot t_7 + 0 \cdot t_8 + 0 \cdot t_9 = t_{s1} + t_{s4} \\ -t_1 + 4 \cdot t_2 - t_3 + 0 \cdot t_4 - t_5 + 0 \cdot t_6 + 0 \cdot t_7 + 0 \cdot t_8 + 0 \cdot t_9 = t_{s4} \\ 0 \cdot t_1 - t_2 + 4 \cdot t_3 + 0 \cdot t_4 + 0 \cdot t_5 - t_6 + 0 \cdot t_7 + 0 \cdot t_8 + 0 \cdot t_9 = t_{s3} + t_{s4} \\ -t_1 + 0 \cdot t_2 + 0 \cdot t_3 + 4 \cdot t_4 - t_5 + 0 \cdot t_6 - t_7 + 0 \cdot t_8 + 0 \cdot t_9 = t_{s1} \\ 0 \cdot t_1 - t_2 + 0 \cdot t_3 - t_4 + 4 \cdot t_5 - t_6 + 0 \cdot t_7 - t_8 + 0 \cdot t_9 = 0 \\ 0 \cdot t_1 + 0 \cdot t_2 - t_3 + 0 \cdot t_4 - t_5 + 4 \cdot t_6 + 0 \cdot t_7 + 0 \cdot t_8 - t_9 = t_{s3} \\ 0 \cdot t_1 + 0 \cdot t_2 + 0 \cdot t_3 - t_4 + 0 \cdot t_5 + 0 \cdot t_6 + 4 \cdot t_7 - t_8 + 0 \cdot t_9 = t_{s1} + t_{s2} \\ 0 \cdot t_1 + 0 \cdot t_2 + 0 \cdot t_3 + 0 \cdot t_4 - t_5 + 0 \cdot t_6 - t_7 + 4 \cdot t_8 - t_9 = t_{s2} \\ 0 \cdot t_1 + 0 \cdot t_2 + 0 \cdot t_3 + 0 \cdot t_4 + 0 \cdot t_5 - t_6 + 0 \cdot t_7 - t_8 + 4 \cdot t_9 = t_{s2} + t_{s3} \end{array} \right. \quad (10)$$

Taking into account the fact that the Solver instrument from Microsoft Excel® operates with a certain group of cells which are correlated with the objective cell (Set Objective), and after the numerical procedure it can modify the values from these cells indicated by the user, the unknown temperature values $t_1 \dots t_9$ are initialized as shown in Figure 4, using for example the value 1.0°C .

Implementare in Excel									
	t1	t2	t3	t4	t5	t6	t7	t8	t9
Valori variabile	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Fig. 4. Initializing the unknown temperatures $t_1 \dots t_9$ in the nodes of the square mesh

For the linear equation system (10), the usual matrix form (11) was used [1], [2], [3] etc.:

$$CM \times VCV = CCV \quad (11)$$

where:

- CM – The coefficient matrix (12);
- VCV – The variable column vector (13);
- CCV – The constant column vector (14).

$$CM = \begin{bmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 4 & -1 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 4 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 4 \end{bmatrix} \quad (12)$$

$$VCV = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \\ t_7 \\ t_8 \end{bmatrix} \quad (13)$$

$$CCV = \begin{bmatrix} t_{s1} + t_{s4} \\ t_{s4} \\ t_{s3} + t_{s4} \\ t_{s1} \\ 0 \\ t_{s3} \\ t_{s1} + t_{s2} \\ t_{s2} \\ t_{s2} + t_{s3} \end{bmatrix} \quad (14)$$

The coefficient matrix (CM) is implemented in the spreadsheet as in Figure 5.

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Ecuatia1	4.0	-1.0	0.0	-1.0	0.0	0.0	0.0	0.0	0.0
Ecuatia2	-1.0	4.0	-1.0	0.0	-1.0	0.0	0.0	0.0	0.0
Ecuatia3	0.0	-1.0	4.0	0.0	0.0	-1.0	0.0	0.0	0.0
Ecuatia4	-1.0	0.0	0.0	4.0	-1.0	0.0	-1.0	0.0	0.0
Ecuatia5	0.0	-1.0	0.0	-1.0	4.0	-1.0	0.0	-1.0	0.0
Ecuatia6	0.0	0.0	-1.0	0.0	-1.0	4.0	0.0	0.0	-1.0
Ecuatia7	0.0	0.0	0.0	-1.0	0.0	0.0	4.0	-1.0	0.0
Ecuatia8	0.0	0.0	0.0	0.0	-1.0	0.0	-1.0	4.0	-1.0
Ecuatia9	0.0	0.0	0.0	0.0	0.0	-1.0	0.0	-1.0	4.0

Fig. 5. Implementing the coefficient matrix (CM)

The initial values of the temperatures $t_1 \dots t_9$ from Figure 4 and the CM (12) associated to the LES were used to calculate a column of initial results (the column called **Rezultat** from Figure 6).

This column has to be compared term by term, by using the operator **Op** equal (=), with the corresponding numerical values from CCV (the column called **Problema** from Figure 6) which are obtained using the temperatures $t_{s1} \dots t_{s4}$ on the delimiting surfaces of the square cross section.

Rezultat	Op.	Problema
2.0	=	200.0
1.0	=	50.0
2.0	=	100.0
1.0	=	150.0
0.0	=	0.0
1.0	=	50.0
2.0	=	200.0
1.0	=	50.0
2.0	=	100.0

Fig. 6. The initial results (column **Rezultat**) and the numerical values from CCV (column **Problema**)

After numerically solving the LES (10) with nine unknowns, the final values obtained in the column **Rezultat** must be identical with those from the column **Problema** associated to CCV (14).

As already mentioned in §1, because the Solver instrument operates with a cell called objective (Set Objective), in order to point to that cell, it is necessary to use inside the spreadsheet an additional line called **Condiție**, which contains this cell.

For this purpose, any equation from LES (10) may be used, so for example in Figure 7, equation no. 1 was used and emphasized.

Condiție	4.0	-1.0	0.0	-1.0	0.0	0.0	0.0	0.0	0.0	2.0	=	200.0
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Fig. 7. Using the condition that contains the objective cell (equation no. 1 from LES)

The final form of the computer implementation of the proposed problem in §2 is indicated in Figure 8 and contains all the elements that were previously and individually

presented in Figures 4, 5, 6 and 7: initializing the unknown temperatures $t_1 \dots t_9$, the coefficient matrix (CM), the column of the initial results, the constant column vector (CCV) and the additional line containing the necessary condition for operating the Solver instrument.

Implementare in Excel												
	t1	t2	t3	t4	t5	t6	t7	t8	t9	Rezultat	Op.	Problema
Valori variabile	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000			
Ecuatia1	4.0	-1.0	0.0	-1.0	0.0	0.0	0.0	0.0	0.0	2.0	=	200.0
Ecuatia2	-1.0	4.0	-1.0	0.0	-1.0	0.0	0.0	0.0	0.0	1.0	=	50.0
Ecuatia3	0.0	-1.0	4.0	0.0	0.0	-1.0	0.0	0.0	0.0	2.0	=	100.0
Ecuatia4	-1.0	0.0	0.0	4.0	-1.0	0.0	-1.0	0.0	0.0	1.0	=	150.0
Ecuatia5	0.0	-1.0	0.0	-1.0	4.0	-1.0	0.0	-1.0	0.0	0.0	=	0.0
Ecuatia6	0.0	0.0	-1.0	0.0	-1.0	4.0	0.0	0.0	-1.0	1.0	=	50.0
Ecuatia7	0.0	0.0	0.0	-1.0	0.0	0.0	4.0	-1.0	0.0	2.0	=	200.0
Ecuatia8	0.0	0.0	0.0	0.0	-1.0	0.0	-1.0	4.0	-1.0	1.0	=	50.0
Ecuatia9	0.0	0.0	0.0	0.0	0.0	-1.0	0.0	-1.0	4.0	2.0	=	100.0
Conditie	4.0	-1.0	0.0	-1.0	0.0	0.0	0.0	0.0	0.0	2.0	=	200.0

Fig. 8. The final form of the computer implementation used for solving the problem

The calculation of the temperature values $t_1 \dots t_9$ in the mesh nodes from Figure 2 is being performed numerically, by solving the LES (10) using the Solver instrument, according to the following steps:

- a) From the “Data” ribbon of Microsoft Excel®, it is necessary to call the dialog window “Solver Parameters” (Figure 10) of the Solver instrument.

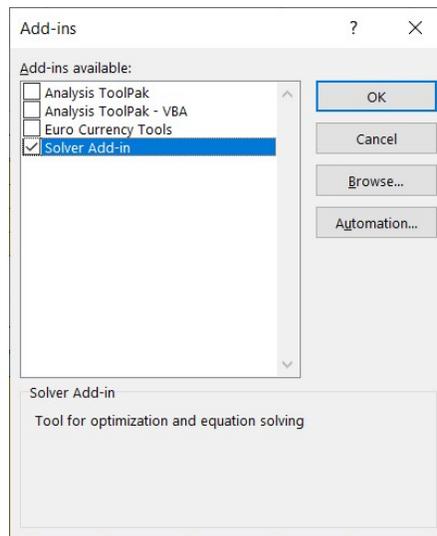


Fig. 9. Installing the Solver instrument as an add-in

Note:

The Solver instrument is an add-in that must be installed before its first use. The installation is made by calling from the menu *File -> Options -> Add-ins -> Excel Add-ins -> Go ...* the “Add-ins” dialog window presented in Figure 9, in which the validation option “Solver Add-in” must be checked and then the OK button must be pressed;

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- b) Give the reference to the objective cell (\$L\$43 in Figure 10) contained in the additional line called **Conditione**, check the option “Value Of”, then in the text box nearby give the value that must be obtained. Using equation no. 1 of LES (10), the necessary value is the first corresponding value from CCV (14), as shown for example in Figure 8;
- c) The cells containing the unknown temperature values $t_1 \dots t_9$ initialized with the value of 1.0°C in Figure 4 must be indicated directly or by selecting them with the current pointing device (mouse) inside the “By Changing Variable Cells” box. It is recommended that these cells should form a contiguous range in the spreadsheet e.g., \$C\$33:\$K\$33 as in Figure 10;
- d) Inside the “Subject to the Constraints” list box, it is necessary to give the restrictions (constrains) that are defining the solution of the problem, using the dedicated buttons (“Add”, ”Change”, ”Delete”) available in the ”Solver Parameters” dialog window. These restrictions are defined by the fact that the correct results are obtained when the numerical values from the column **Rezultat** become identical to those from the column **Problema** (Figure 8);

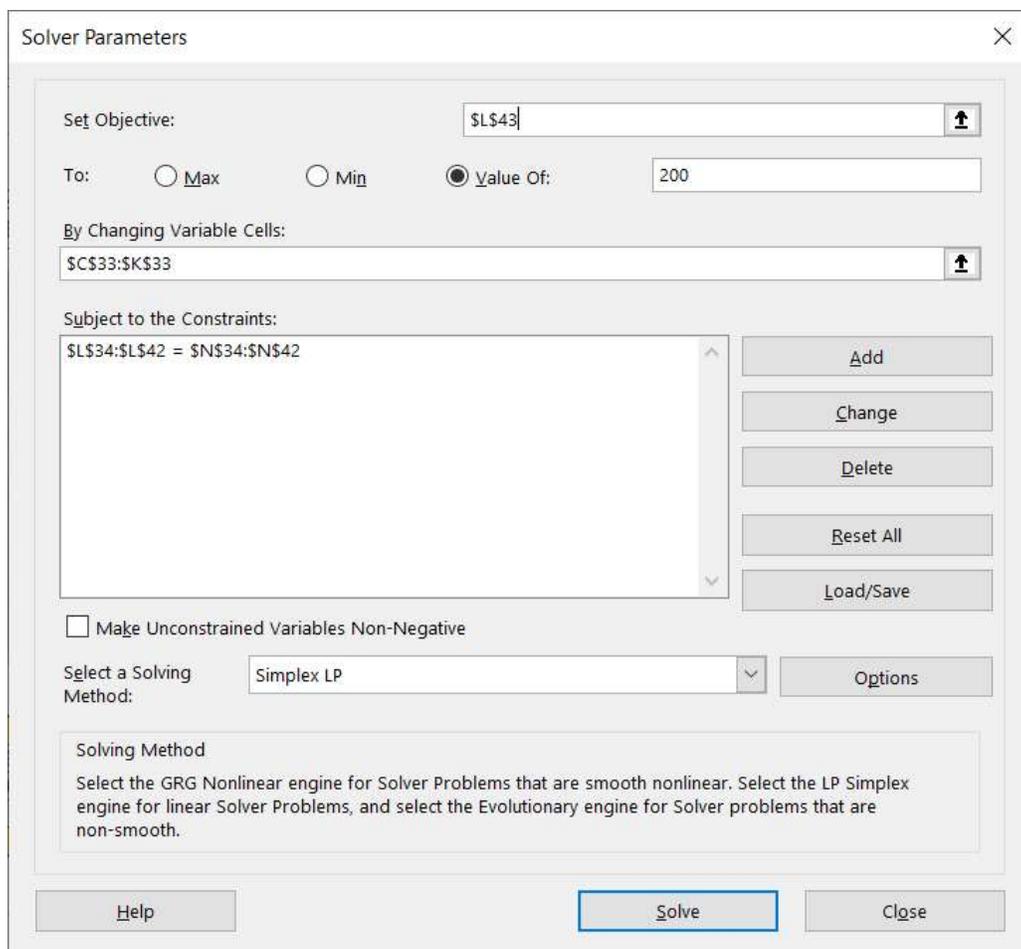


Fig. 10. Specifying solving parameters used by Solver

- e) Optionally, other advanced solving preferences may be specified by using the “Options” button;
- f) Finally, the “Solve” button must be pressed.

The numerical values of the temperatures $t_1 \dots t_9$ in the nodes of the chosen square mesh, obtained by solving the LES (10) numerically and modified by Solver in the cells indicated by the user in the “By Changing Variable Cells” box, are presented in Figure 11.

Rezultate	t1	t2	t3	t4	t5	t6	t7	t8	t8	Rezultat	Op.	Problema
Valori variabile	92.857	68.750	57.143	102.679	75.000	59.821	92.857	68.750	57.143			
Ecuatia1	4.0	-1.0	0.0	-1.0	0.0	0.0	0.0	0.0	0.0	200.0	=	200.0
Ecuatia2	-1.0	4.0	-1.0	0.0	-1.0	0.0	0.0	0.0	0.0	50.0	=	50.0
Ecuatia3	0.0	-1.0	4.0	0.0	0.0	-1.0	0.0	0.0	0.0	100.0	=	100.0
Ecuatia4	-1.0	0.0	0.0	4.0	-1.0	0.0	-1.0	0.0	0.0	150.0	=	150.0
Ecuatia5	0.0	-1.0	0.0	-1.0	4.0	-1.0	0.0	-1.0	0.0	0.0	=	0.0
Ecuatia6	0.0	0.0	-1.0	0.0	-1.0	4.0	0.0	0.0	-1.0	50.0	=	50.0
Ecuatia7	0.0	0.0	0.0	-1.0	0.0	0.0	4.0	-1.0	0.0	200.0	=	200.0
Ecuatia8	0.0	0.0	0.0	0.0	-1.0	0.0	-1.0	4.0	-1.0	50.0	=	50.0
Ecuatia9	0.0	0.0	0.0	0.0	0.0	-1.0	0.0	-1.0	4.0	100.0	=	100.0
Conditie	4.0	-1.0	0.0	-1.0	0.0	0.0	0.0	0.0	0.0	200.0	=	200.0

Fig. 11. Numerical values of the temperatures $t_1 \dots t_9$

In order to obtain an overall numerical image of the 2D temperature field for the square cross section, the temperature values $t_1 \dots t_9$ in the mesh nodes, the temperatures $t_{s1} \dots t_{s4}$ on the delimiting surfaces of the section and the temperature values in its corners, may be grouped together in an uncluttered region of the spreadsheet, as shown in Figure 12.

100	150	150	150	100
50	92.857	102.679	92.857	50
50	68.750	75.000	68.750	50
50	57.143	59.821	57.143	50
50	50	50	50	50

Fig. 12. The 2D temperature field in the square cross section (numerical representation)

If there is no additional information or technical specifications available regarding the nodes situated at the intersection of two surfaces having constant temperature (the corners of the cross section), the temperature in these nodes may be estimated by using the arithmetic average of the respective surface temperatures (13) [1] etc.:

$$Node\ m - n: t_j = \frac{t_{s,m} + t_{s,n}}{2} \tag{15}$$

The numerical image of the 2D temperature field in the square cross section (Figure 12) can be graphically translated, by using one or more methods of graphical representation that Microsoft Excel® gives to the user. For example, in Figure 13 is presented a 2D graphical image of the temperature field, using interpolated contours, and in Figure 14 is showed a 3D image of the temperature values from the field.

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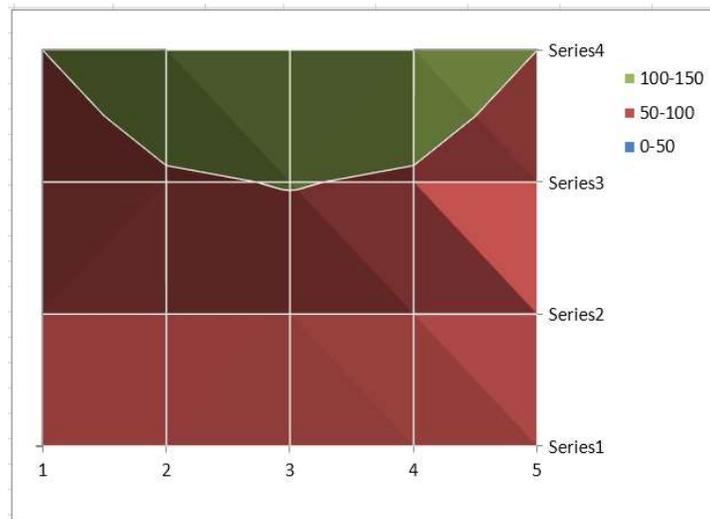


Fig. 13. Graphical representation of the 2D temperature field in the square cross section, using interpolated contours

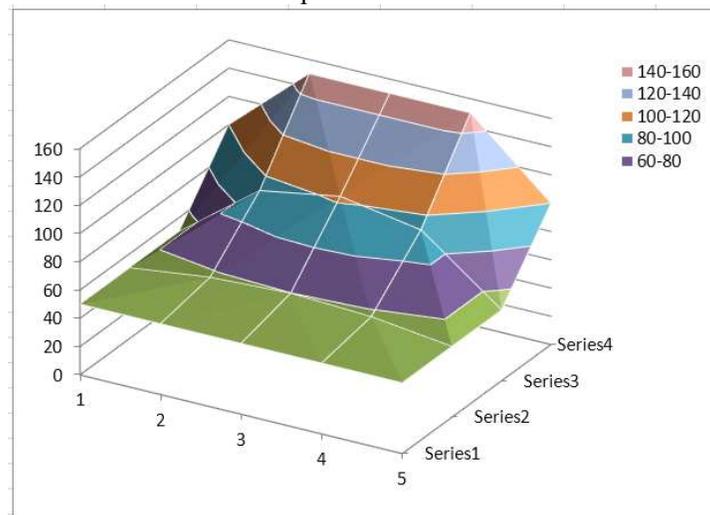


Fig. 14. 3D graphical representation of the temperature values from the square cross section field

3. Studied and solved cases

The presented software instrument solves numerically the application proposed in this paper, but its utility is proven especially in the cases of analysis of the 2D temperature field for the square cross section, when modifications of the initial temperature values $t_{s1} \dots t_{s4}$ on the delimiting surfaces of the section appear.

With an additional effort for implementing, the instrument can be adapted to solve a more complicated LES, by using a „finer” discretization mesh, having a smaller spacing step for temperature nodal points generation.

In the cases of study and analysis of the 2D temperature field for the square cross section, the temperature values $t_{s1} \dots t_{s4}$ are modified in the corresponding cells in Figure 3. Later, the Solver instrument will solve the LES (10), using the same parameters as

those indicated in Figure 10 and adapting the value from the text box situated on the right of the “Value Of” control to the studied case.

Some of the analyzed situations are presented hereinafter, by indicating in each case: the initial data, the solved LES, the numerical results obtained and the graphical representations describing the temperature field of the square cross section.

Case no. 1

Temperatures on the delimiting surfaces of the square cross section:

$$\begin{cases} t_{s1} = 80^{\circ}\text{C} \\ t_{s2} = 30^{\circ}\text{C} \\ t_{s3} = 80^{\circ}\text{C} \\ t_{s4} = 30^{\circ}\text{C} \end{cases}$$

Marime	ID	U.M.	Valoare
Temp. suprafata 1	ts1	80	C
Temp. suprafata 2	ts2	30	C
Temp. suprafata 3	ts3	80	C
Temp. suprafata 4	ts4	30	C

Fig. 15. Case no. 1: Initial data

Rezultate	t1	t2	t3	t4	t5	t6	t7	t8	t8	Rezultat	Op.	Problema
Valori variabile	55.000	48.750	55.000	61.250	55.000	61.250	55.000	48.750	55.000			
Ecuatia1	4.0	-1.0	0.0	-1.0	0.0	0.0	0.0	0.0	0.0	110.0	=	110.0
Ecuatia2	-1.0	4.0	-1.0	0.0	-1.0	0.0	0.0	0.0	0.0	30.0	=	30.0
Ecuatia3	0.0	-1.0	4.0	0.0	0.0	-1.0	0.0	0.0	0.0	110.0	=	110.0
Ecuatia4	-1.0	0.0	0.0	4.0	-1.0	0.0	-1.0	0.0	0.0	80.0	=	80.0
Ecuatia5	0.0	-1.0	0.0	-1.0	4.0	-1.0	0.0	-1.0	0.0	0.0	=	0.0
Ecuatia6	0.0	0.0	-1.0	0.0	-1.0	4.0	0.0	0.0	-1.0	80.0	=	80.0
Ecuatia7	0.0	0.0	0.0	-1.0	0.0	0.0	4.0	-1.0	0.0	110.0	=	110.0
Ecuatia8	0.0	0.0	0.0	0.0	-1.0	0.0	-1.0	4.0	-1.0	30.0	=	30.0
Ecuatia9	0.0	0.0	0.0	0.0	0.0	-1.0	0.0	-1.0	4.0	110.0	=	110.0
Conditie	4.0	-1.0	0.0	-1.0	0.0	0.0	0.0	0.0	0.0	110.0	=	110.0

Fig. 16. Case no. 1: Solved LES

55	80	80	80	55
30	55.000	61.250	55.000	30
30	48.750	55.000	48.750	30
30	55.000	61.250	55.000	30
55	80	80	80	55

Fig. 17. Case no. 1: Numerical representation of the 2D temperature field

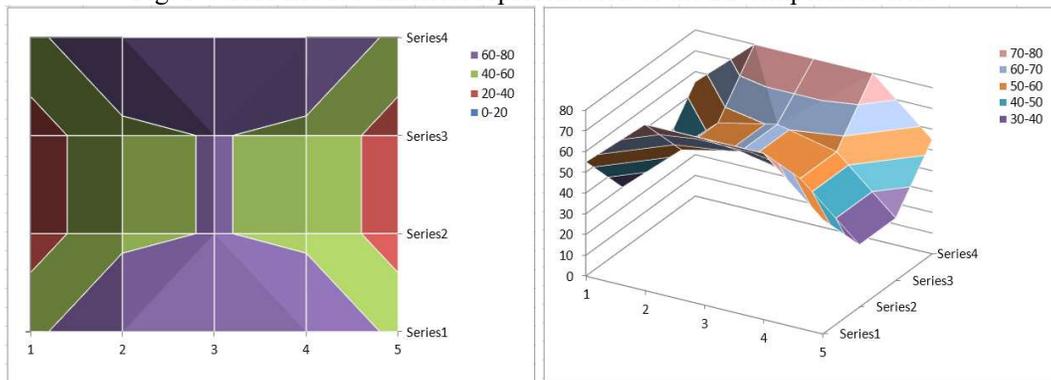


Fig. 18. Case no. 1: 2D and 3D graphical representation of the temperature field

Steady state two-dimensional heat conduction in a square cross section, by using Microsoft Excel® for numerical modeling

Case no. 2

Temperatures on the delimiting surfaces of the square cross section:

$$\begin{cases} t_{s1} = 45^{\circ}\text{C} \\ t_{s2} = 70^{\circ}\text{C} \\ t_{s3} = 20^{\circ}\text{C} \\ t_{s4} = 50^{\circ}\text{C} \end{cases}$$

Marime	ID	U.M.	Valoare
Temp. suprafata 1	ts1	45	C
Temp. suprafata 2	ts2	70	C
Temp. suprafata 3	ts3	20	C
Temp. suprafata 4	ts4	50	C

Fig. 19. Case no. 2: Initial data

Rezultate	t1	t2	t3	t4	t5	t6	t7	t8	t8	Rezultat	Op.	Problema
Valori variabile	47.143	45.402	38.214	48.170	46.250	37.455	54.286	53.973	45.357			
Ecuatia1	4.0	-1.0	0.0	-1.0	0.0	0.0	0.0	0.0	0.0	95.0	=	95.0
Ecuatia2	-1.0	4.0	-1.0	0.0	-1.0	0.0	0.0	0.0	0.0	50.0	=	50.0
Ecuatia3	0.0	-1.0	4.0	0.0	0.0	-1.0	0.0	0.0	0.0	70.0	=	70.0
Ecuatia4	-1.0	0.0	0.0	4.0	-1.0	0.0	-1.0	0.0	0.0	45.0	=	45.0
Ecuatia5	0.0	-1.0	0.0	-1.0	4.0	-1.0	0.0	-1.0	0.0	0.0	=	0.0
Ecuatia6	0.0	0.0	-1.0	0.0	-1.0	4.0	0.0	0.0	-1.0	20.0	=	20.0
Ecuatia7	0.0	0.0	0.0	-1.0	0.0	0.0	4.0	-1.0	0.0	115.0	=	115.0
Ecuatia8	0.0	0.0	0.0	0.0	-1.0	0.0	-1.0	4.0	-1.0	70.0	=	70.0
Ecuatia9	0.0	0.0	0.0	0.0	0.0	-1.0	0.0	-1.0	4.0	90.0	=	90.0
Conditie	4.0	-1.0	0.0	-1.0	0.0	0.0	0.0	0.0	0.0	95.0	=	95.0

Fig. 20. Case no. 2: Solved LES

47.5	45	45	45	57.5
50	47.143	48.170	54.286	70
50	45.402	46.250	53.973	70
50	38.214	37.455	45.357	70
35	20	20	20	45

Fig. 21. Case no. 2: Numerical representation of the 2D temperature field

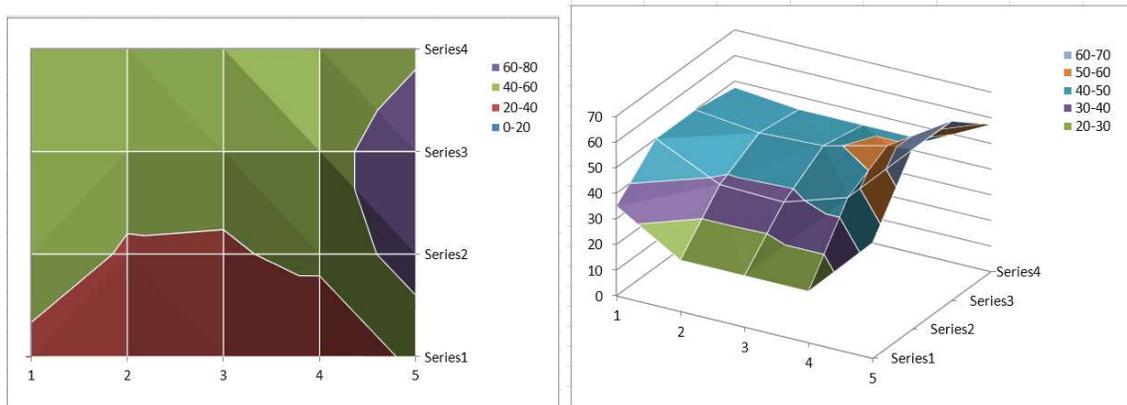


Fig. 22. Case no. 2: 2D and 3D graphical representation of the temperature field

Case no. 3

Temperatures on the delimiting surfaces of the square cross section:

$$\begin{cases} t_{s1} = 50^{\circ}\text{C} \\ t_{s2} = 25^{\circ}\text{C} \\ t_{s3} = 0^{\circ}\text{C} \\ t_{s4} = 10^{\circ}\text{C} \end{cases}$$

Marime	ID	U.M.	Valoare
Temp. suprafata 1	ts1	50	C
Temp. suprafata 2	ts2	25	C
Temp. suprafata 3	ts3	0	C
Temp. suprafata 4	ts4	10	C

Fig. 23. Case no. 3: Initial data

Rezultate	t1	t2	t3	t4	t5	t6	t7	t8	t8	Rezultat	Op.	Problema
Valori variabile	27.500	17.098	9.643	32.902	21.250	11.473	32.857	23.527	15.000			
Ecuatia1	4.0	-1.0	0.0	-1.0	0.0	0.0	0.0	0.0	0.0	60.0	=	60.0
Ecuatia2	-1.0	4.0	-1.0	0.0	-1.0	0.0	0.0	0.0	0.0	10.0	=	10.0
Ecuatia3	0.0	-1.0	4.0	0.0	0.0	-1.0	0.0	0.0	0.0	10.0	=	10.0
Ecuatia4	-1.0	0.0	0.0	4.0	-1.0	0.0	-1.0	0.0	0.0	50.0	=	50.0
Ecuatia5	0.0	-1.0	0.0	-1.0	4.0	-1.0	0.0	-1.0	0.0	0.0	=	0.0
Ecuatia6	0.0	0.0	-1.0	0.0	-1.0	4.0	0.0	0.0	-1.0	0.0	=	0.0
Ecuatia7	0.0	0.0	0.0	-1.0	0.0	0.0	4.0	-1.0	0.0	75.0	=	75.0
Ecuatia8	0.0	0.0	0.0	0.0	-1.0	0.0	-1.0	4.0	-1.0	25.0	=	25.0
Ecuatia9	0.0	0.0	0.0	0.0	0.0	-1.0	0.0	-1.0	4.0	25.0	=	25.0
Conditie	4.0	-1.0	0.0	-1.0	0.0	0.0	0.0	0.0	0.0	60.0	=	60.0

Fig. 24. Case no. 3: Solved LES

30	50	50	50	37.5
10	27.500	32.902	32.857	25
10	17.098	21.250	23.527	25
10	9.643	11.473	15.000	25
5	0	0	0	12.5

Fig. 25. Case no. 3: Numerical representation of the 2D temperature field

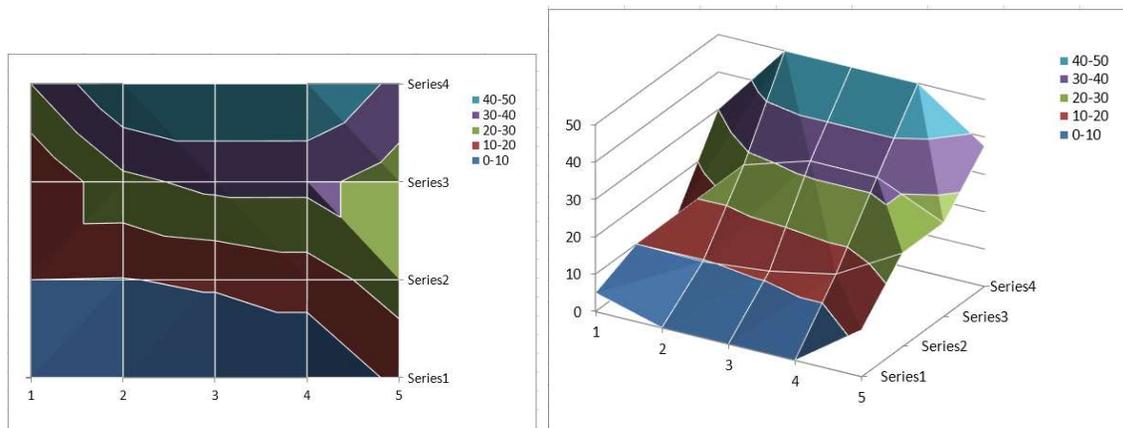


Fig. 26. Case no. 3: 2D and 3D graphical representation of the temperature field

4. Conclusions

The problem of steady state heat transfer in a square cross section may be numerically solved by using the finite differences method (FDM). The differential

equation (2) that characterizes the 2D temperature field in the section is replaced by the finite differences equation (3), then a squared discretization mesh with an equal spacing step $\Delta x = \Delta y = \Delta$ for node generation on both Ox and Oy coordinate axes is used, and finally the unknown temperatures in the mesh nodes are expressed using the relation (7).

The resulting linear equation system LES (9) is numerically solved by calling the Solver instrument from Microsoft Excel® software, part of the Microsoft Office® suite. This instrument can modify the initial values indicated by the user in certain cells of the spreadsheet, with the purpose to later obtain the targeted result in the objective cell (Set Objective).

The preparations for solving the studied problem were made in several steps: introduction of initial data as separate variables (Figure 3), initialization of the unknown variables (Figure 4), description of LES (9) in a matrix form (10), successive implementation of the coefficient matrix CM (Figure 5), the constant column vector CCV (Figure 6) and an additional line called Condition (Figure 7). The final form of the computer implementation of the problem-solving process is presented in Figure 8.

The numerical solving of the LES (9) was done by using the Solver instrument, specifying the necessary solving parameters: the reference to the objective cell and the target value, respectively, the group of cells containing the values of the unknown temperatures, the solving restrictions (constraints) and finally the choice for the SimplexLP algorithm as a preferred method for problem-solving (Figure 10).

The obtained numerical results regarding the values of the unknown temperatures in the square section field, the temperatures on its delimiting surfaces and the estimated temperatures in the corners of the section were grouped together in Figure 12 in order to offer an overall numerical image of the 2D temperature field of the section. For a more intuitive visualization, these numerical values may be graphically depicted using the various 2D or 3D types of representations that Microsoft Excel® offers to the user (Figure 13 and Figure 14).

A series of cases were studied and solved, when temperatures on the delimiting surfaces of the square section were modified, generating a certain thermal load, symmetric or asymmetric. Using the separate variable implementation of those temperature values, the modifications were done without any additional complications, and then by calling the Solver instrument, the numerical and graphical results corresponding to each case of study were quickly obtained.

In order to analyze some more detailed 2D temperature fields, the created software instrument can be adapted with an additional implementation effort, by modifying the spacing step for the positioning of the mesh nodes and by increasing the number of unknown temperatures i.e., the number of nodes. However, this strategy will also increase the number of equations in the LES, and consequently, the complexity of the problem that has to be solved. In this case when the number of nodes and temperatures within, respectively, will become very large, it is preferable to better use a dedicated computer software, that will automatically build the discretization mesh and solve the generated LES.

The instrument for the study and automatic calculation of the 2D temperature field in a square cross section that was presented in this paper is useful for both the students of the Building Services Faculty studying the multidimensional heat transfer, and the domain specialists interested in practical solving these types of engineering applications.

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