Revista Română de Inginerie Civilă, Volumul 5 (2014), Numărul 2

Heron's fountain demonstrator*

Andrei-Mugur Georgescu[1], Sanda-Carmen Georgescu[2], Liviu Stroia[3]

[1]Technical University of Civil Engineering Bucharest
B-dul. Lacul Tei 124, sect. 2, 020396 Bucharest, Romania.
E-mail: andreig@utcb.ro

[2]University Politehnica of Bucharest
Splaiul Independentei 313, sect. 6, 060042 Bucharest, Romania.
E-mail: carmen.georgescu@upb.ro

[3]Sangari Engineering Services
Str. Emil Racovita 35-39, Complex Azur 1, vila AP08, 077191 Voluntari, Ilfov, Romania.
E-mail: liviu.stroia@sangari.ro

Abstract. *Heron's fountain has been described by Heron of Alexandria in his treatise "The Pneumatics". In antiquity, scientists didn't state physical principles governing fluid motion as we do now, but they understood how things worked. We found similarities between Heron's fountain and some modern systems. Heron's original design was modified to make those similarities more obvious, and allow a more continuous use of the fountain. In this paper, we will focus on an experimental setup, which exemplifies the principles of non-isothermal flow in central heating systems.*

Key words: Heron's fountain, non-isothermal flow, central heating system

1. Introduction

Physical principles governing the motion of fluids are unchanged since the creation of the Universe. In Ancient times, scientists didn't state those principles as we use to do now, and it is often hard for us to understand what they were thinking about one thing or another. This doesn't mean they didn't understood how things worked.

Nowadays, when new technologies, as well as computational fluid dynamics software are fully available in one mouse click, a major challenge in teaching a general Hydraulics course to engineers is to keep them focused on the physical principles that lay behind the studied phenomenon. To achieve such a goal, one has to use the oldest tricks in the book (a Hydraulics book of course). In this paper, the book in question is *The Pneumatics* by Heron of Alexandria [1], and the "trick" is *Heron's Fountain*.

For the modern reader, the title of the above cited treatise may seem unrelated to liquids, the modern meaning of the word being "the use of compressed gases to affect mechanical motion". In Antiquity, the term was given various technical meanings by medical writers, philosophers, mathematicians, or engineers. When reading Heron's treatise, we notice that from the 78 sections included, only one deals

* Lucrare inclusă în programul conferinţei EENVIRO 2013

exclusively with air flow. The other 77 sections deal with liquids (mostly water, but sometimes also wine), or both liquids and air.

We used the term "trick" for Heron's Fountain only because there are no known practical applications of this device in Antiquity, except (perhaps) for rich people to "show off" before their guests, with a fountain that works (for some time) without any external intervention.

In this paper, we will focus on an experimental setup, derived from Heron's Fountain, that we successfully used to familiarize students with the principles of non-isothermal flow in central heating systems.

2. Heron's Fountain

Here is the reminder of Heron's description of the fountain (see Figure 1), extracted from *The Pneumatics* by Heron of Alexandria [1, Section 36]:
"Construct on a pedestal the figure of a Satyr holding in his hands a wine-skin: place near a washing-basin, and into this let some liquid be poured until it is full; water shall be made to flow into the basin without running over, until all the water in the skin is exhausted. The following is the construction. Let A B (Fig. 1), be a perfectly airtight pedestal, either cylindrical or octagonal in shape, as may seem more elegant, and divided into two chambers by the partition C D, through which the tube E F fitting closely into the partition, extends upwards nearly to the roof of the pedestal. Through the roof insert the tube G H, projecting slightly above the vessel, and lying exactly under the basin, while, below, it reaches to the bottom except that room must be left for the passage of water: this tube must be soldered into the roof of the pedestal and the partition. Another tube, K L M, must also be inserted through the roof, reaching not quite so low as the partition, soldered into the roof and carrying its stream into the basin, which lies above the tube G H and communicates with it. Now let the vessel A D be filled with water through an orifice N, which must be afterwards closed. If water is poured into the basin, it will pass through the tube G H into the vessel B C; and the air in B C, passing through the tube E F and into the vessel A D, will force the liquid in A D through K L M into the basin; and this being carried again into B C will force out the contained air as before, which, again, will force the water in the vessel A D into the basin: and this will go on until the water in A D is exhausted. The tube K L M must pass through the mouth of the skin and be particularly fine, that the display may last a considerable time."

Nowadays, this description seems not that technical and rather simple. In fact, it's all one needs to build such a fountain. Basically, there are 3 tanks positioned one above the other (see Figure 2). The upper tank is opened to the atmosphere, while the other two tanks are under pressure. At the start, the mid closed tank is filled with water, and the other tanks are empty (i.e. filled with air). Some water poured in the upper opened tank flows to the lower closed tank, and begins to fill it from the bottom, thus chasing the air it contained towards the top of the mid tank. As air gets into the mid tank, it chases the water it contained towards the upper opened tank, forming a rising jet. The jet returns to the opened tank, so water continues to flow towards the lower tank. The flow stops when the lower tank gets filled with water, while the mid tank is only filled with air, not being able to feed in water the upper opened tank. The problem is that the fountain works on its own, without the use of any apparent source of energy to generate the flow, but only for a limited amount of time. After the flow has stopped, in order to restart it, one must refill with water the mid tank and fill with air the lower tank.

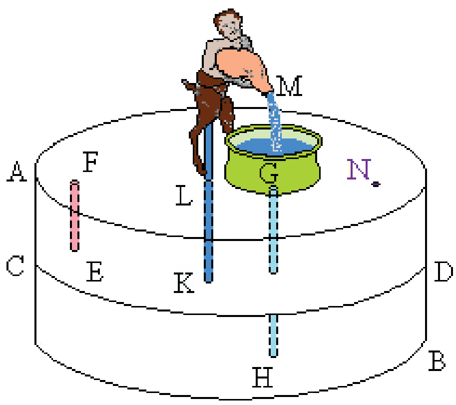


Fig. 1. Heron's Fountain, based on the original drawing presented in [1, Section 36].

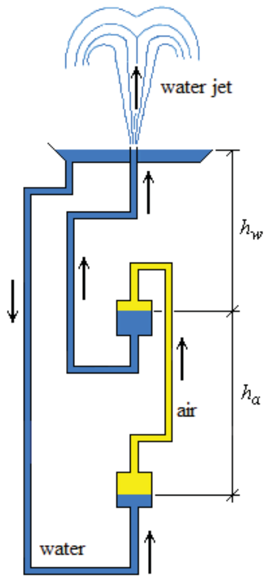


Fig. 2. Hydraulic circuit of Heron's Fountain.

The flow is assured by the difference in pressure existing between the left column $p_{left} = \rho_w g(h_w + h_a)$, and the right column $p_{right} = g(\rho_w h_w + \rho_a h_a)$, as:

$$\Delta p = (\rho_w - \rho_a) g h_a \quad (1)$$

where ρ_w is the density of water, ρ_a is the density of air, and h_a is the height of the air column between the two closed tanks, as in Figure 2. This pressure difference is a function of the height between the two sealed tanks. The level at which the opened tank is positioned has no relevance whatsoever in the pressure difference.

3. Binary hydraulic networks

Binary hydraulic networks are looped networks without active consumers (none of the flowing fluids being actually consumed), i.e. networks in which fluid is used to transport a different physical quantity (usually heat), from a network area, to another [2]. In terms of Hydraulics, the calculations of binary networks are somehow different from usual looped networks [3]. Primarily, due to variations in temperature (T), the fluid can no longer be considered in all respects incompressible, and secondly, values of flow rates and flow directions are imposed in the network by heat exchange considerations.

Temperature variations along the network induce physical variations of liquid parameters: the density $\rho = \rho(T)$ and the dynamic viscosity $\mu = \mu(T)$.

If we consider a closed loop of the network (which is the case of all heating systems using water), i.e. the inlet cross-section is the same as the outlet cross-section ($i \equiv e$), the energy equation simplifies to: $\oint \rho g \, \mathrm{d} z = M_G Q_G^2$, where g is the gravity, z is the elevation, M_G is the gravimetric resistance modulus of the pipe (computed for the average value of the temperature), and Q_G is the gravimetric flow rate (for the full demonstration see [2] or [3]). Thus, the gravimetric flow rate can be computed as:

$$Q_G = \sqrt{\oint \rho g \, \mathrm{d} z / M_G} \quad (2)$$

The relation (2) shows that, in a looped pipe network, there can be a flow rate $Q_G \neq 0$ provided that $\oint \rho g \, \mathrm{d} z \neq 0$ [3]. By examining the closed curve integral, one can see that to realize the above condition, we must have simultaneously: $\rho \neq \text{const}$ and $\mathrm{d} z \neq 0$. This means that for a vertically developed looped network, containing a fluid with variable density along the circuit (due for instance to different temperatures at different points of the network), there exists a flow rate through the pipes, even if there is no hydraulic machinery along the circuit. Obviously, in all central heating systems using water, the density of water is considered to be variable along the system. In the practical case of such heating systems, the flow rate generated by the difference in density is taken into account in the design phase only if: i) the system works only based on this particular flow rate (no hydraulic machineries are used in order to circulate the fluid); ii) the system is high enough to make the difference in density important when pressure is computed at the base of the system. Typically, the height of the system must be over 15 m.

Usually, in order to simplify the calculation, for central heating system using water, the density is considered to have one constant value ρ_t on the outgoing pipes (with higher temperature) from the boiler to the radiators, and another constant value ρ_r on the incoming pipes (with lower temperature) from the radiators to the boiler [4]. For a system vertically developed on a height h. The gravimetric flow rate is considered to be generated by a pressure difference Δp, which is ensured by the difference between the temperatures of the outgoing and incoming pipes, as $Q_G = \sqrt{\Delta p / M_G}$, where

$$\Delta p = \left(\rho_r - \rho_t\right) g h \quad (3)$$

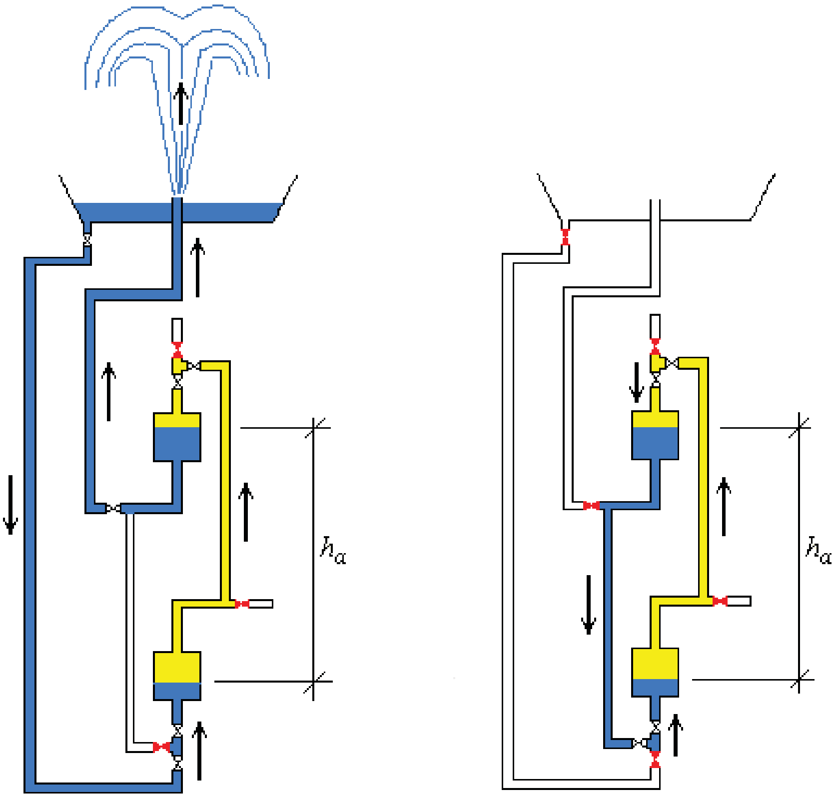
This relation is basically the same as the one of the pressure difference (1) derived for the functioning of Heron's fountain. Of course, in the fountain case, we do not have a single fluid flowing with different densities, but two fluids with different densities. Nevertheless, the pressure difference is given by the difference in densities.

In the following section, we will present Heron's fountain demonstrator and explain the differences with the original design of Heron.

4. Heron’s Fountain demonstrator

The idea that we had was that the “fountain” could also work if it consisted only of the two closed tanks linked together, top to top and bottom to bottom (with the lower tank containing air, and the upper tank containing water). In this case the setup would look very much like a closed loop of a central heating system with the boiler represented by the lower tank (that generates the lower density fluid at its top, by heating it), and the topmost radiator represented by the upper closed tank (that generates at its bottom the higher density fluid, by cooling it).

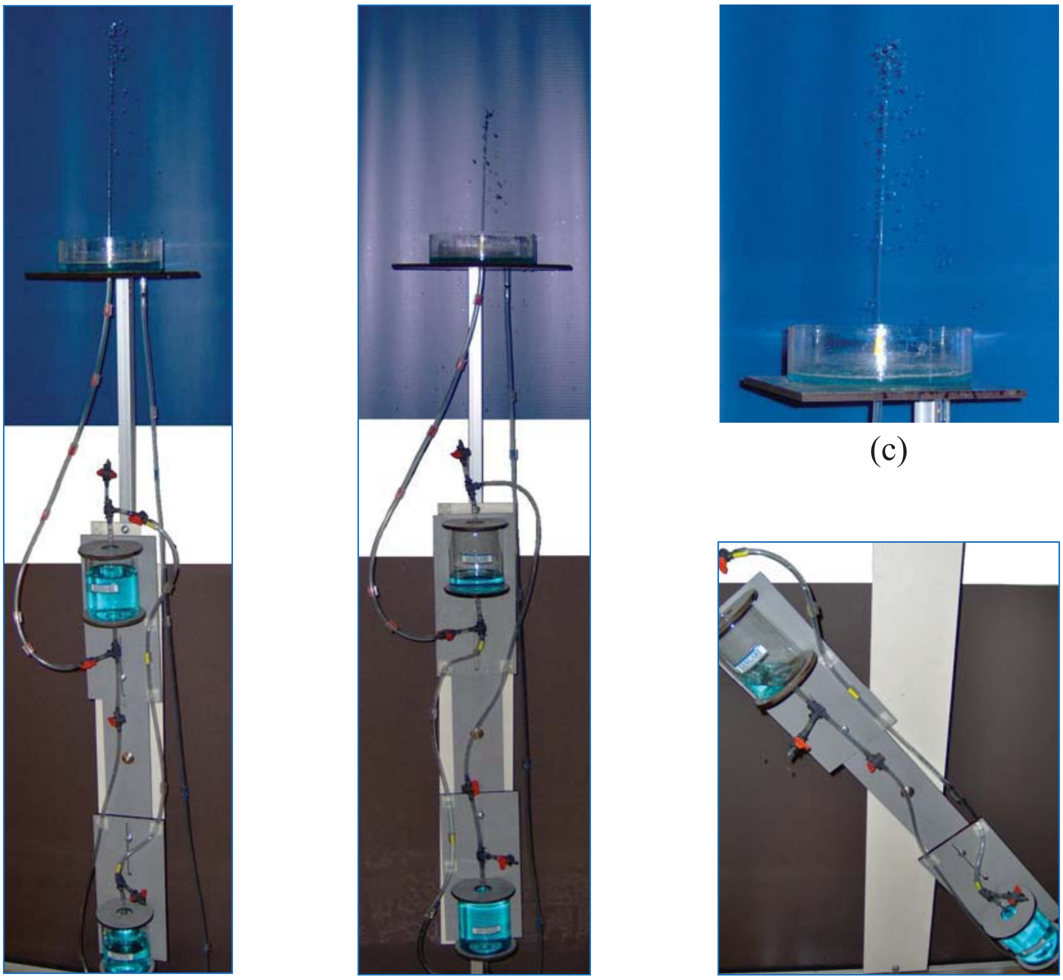
Of course, sincerely speaking, such a setup with only two closed tanks, would look rather boring to students or to any other person to which it would be presented. The original fountain working on its own seems much more appealing for a presentation. As our purpose was to make a eye-catching presentation of the subject, we needed to adapt the setup in order to keep both functionalities available (the original fountain as a *captatio benevolentiae*, and the closed loop between the tanks as an appropriate example). This could be achieved by connecting the 3 tanks with rubber hoses and adding to the system, basically, two pipe tees with corresponding valves (see Figure 3).



(a) (b)

Fig. 3. Experimental setup: (a) working like Heron's Fountain; (b) working in a closed loop. Closed valves are coloured in red, while hoses where there is no water flow are left blank, although they may be filled with water.

Yet another improvement could be made in order to be able to measure something with the closed loop setup (Figure 3b). We could make the height between the two closed tanks adjustable and so, show experimentally that the flow rate is only a function of the difference in density and this said height, denoted h_a.



(c)

(a) (b) (d)

Fig. 4. Experimental setup: (a) full view when working like Heron's Fountain;
(b) full view when working like Heron's Fountain, at a moment prior to the end of the process, when the jet falls suddenly; (c) detail of the upper opened tank and rising liquid jet;
(d) detail of the rotation axis and sliding plates.

There was only one problem left. The fountain (Fig. 3a), or the closed loop (Fig. 3b), works only up to the moment when water fills the lower closed tank. If we wanted to show the students at least both the fountain, and then the closed loop, we would have to refill with water the mid closed tank and with air the lower tank in each of the cases. This would normally take some time and diminish the quality of the presentation. Due to the symmetries of the setup between the two closed tanks, we placed them on a vertical rotating plate with the rotation axis at the mid distance between the two closed tanks. This way, all we have to do to restart the flow is: close the valves; disconnect the two rubber hoses linking the closed tanks to the opened one; turn the vertical plate by 180º; link the rubber hoses to the new corresponding taps;

and reopen the valves to assure functioning as fountain, or in a closed loop. Of course, additional valves and pipe tees are needed to perform such an operation, but it only takes less than a minute or so to restart the flow.

The experimental setup obtained is shown if Figure 4, where one can notice the rotation axis and the sliding plates (Fig. 4d) used to modify the distance between the two closed tanks. When working like Heron's Fountain, the liquid flow rate dependency on the pressure difference is visible by the height of the rising jet, which preserves an almost constant value over the whole process. Towards the end of the process, when the lower closed tank fills with water, the jet decays suddenly (Fig. 4b). The initial value of the pressure difference (1) can be modified by changing the initial value of the height h_a.

5. Experiment

The measurements we performed using the experimental set-up aimed at the demonstrating the validity of the relation between the flow rate and the difference in pressure given by (1). The average flow rate can be measured (knowing the volume of the pressurised tanks) by timing the duration of the phenomenon. The average height between water levels in the two tanks can be simply measured with a measuring tape. Knowing the densities of both water and air, the average difference of pressures can be computed according to equation (1). By performing several measurements for the maximal height between the two tanks, a dependency $Q_G = Q_G(\Delta p)$ was established as: $Q_G = c\sqrt{\Delta p}$, with the constant c representing $c = 1/\sqrt{M_G}$.

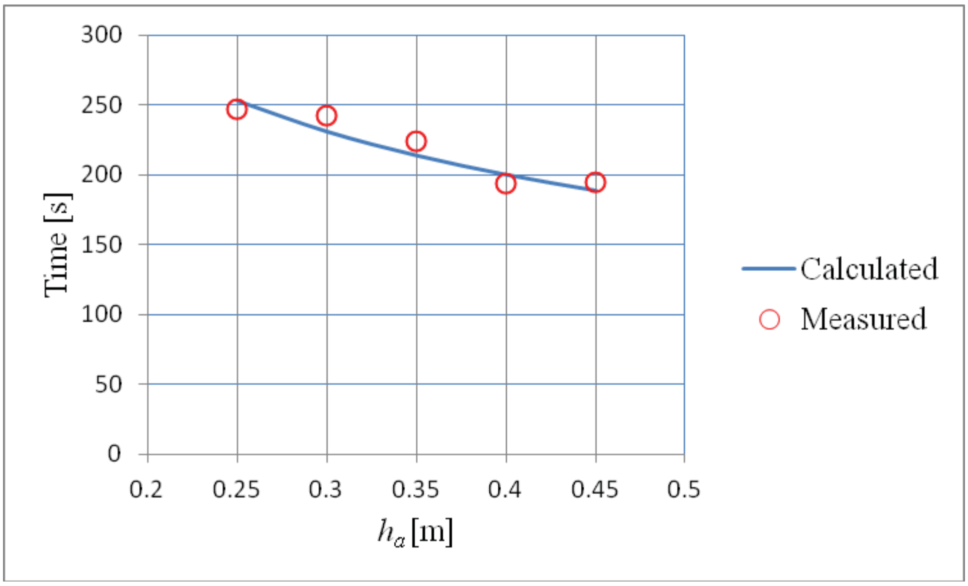


Fig. 5. Calculated and measured values for the duration of the phenomenon the when working like Heron's Fountain at different heights between the two closed tanks.

The value of the constant was then used to compute the flow rate and consequently the duration of the phenomenon for smaller heights. Measurements of the duration of the phenomenon at different heights where then performed and the results were compared with the calculated values. (see Fig. 5). The match between calculated and measured values is very good.

6. Conclusions

After testing the demonstrator several times under different conditions, we consider it’s functioning satisfactory. Working as the original fountain (Heron's fountain) is particularly eye-catching. We believe that such a demonstration of the principle of non-isothermal flow in looped pipe networks can be of great use to teachers on their quest to keep the mind of their students focused on a physical principle and its different (and sometimes apparently unrelated) applications.

Acknowledgements

The authors gratefully acknowledge Sangari Engineering Services – Romania for their help in manufacturing the demonstrator.

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