

# Critic of permeability estimation methods for unsaturated soil

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**Abstract.** *Exploitation of unsaturated soil permeability data ( $k_u$ ) in some cases requires that this characteristic be estimated by methods not measured by laboratory tests and in this case, it is best to use mathematical models to evaluate this parameter.*

*Evaluation of the unsaturated permeability using different estimation methods as well as the studies of comparisons between the results of these models for different types of soils have been treated by several authors ([1], [2]), however none of these authors have compared the results of these models for identical soils (same type of soil, same extraction site, same percentage of different constituents, ect...). In this article we have presented the views of the authors who have treated this subject, then, we applied some models of estimation of  $k_u$  to samples of sand and silty clay. Also we took as a special case, identical samples that have been extracted from the SoilVision 003 database [3].*

*After applying to these samples the models of Kunze & al [4], Campbell [5] and Brooks & Corey [6], we found that all these models can present divergent predictions for samples of the same type of soil and also for identical samples. We deduced therefore that there is no general rule that allows us to choose between  $k_u$  prediction models and that it is always mandatory to go through a series of comparisons in order to select the model that best applies to a soil sample.*

**Keywords:** *unsaturated permeability; SoilVision database, help system, empirical models; statistical models*

## 1. Introduction

Geotechnical design of the ultimate service and ultimate limit state structures is often based on material characteristics measured in the laboratory on samples that are theoretically perfectly saturated with water. However, it appears quickly, in the analysis of measurements carried out, that the saturation in water is rarely total. This is

the case for superficial foundations which rest on unsaturated soils (case of very many constructions).

Permeability of unsaturated soils remains the most difficult parameter to study given its wide range and the number of parameters that influence this characteristic, especially with all the simplifying hypotheses that allowed the development of mathematical models.

For this reason, an in-depth bibliographic study on this subject was carried out and also samples of sand and silty clay were chosen according to well-defined criteria. Models of  $k_u$  predictions have been applied to these different samples. The results of these models were compared to the laboratory results via the calculation of the error presented by each one of these models.

## **2. Critics of direct measurement methods**

Several laboratory tests can be used to measure the permeability coefficient of unsaturated soils [7]. In general, the results obtained are relatively precise. However, because the permeability of unsaturated soils is relatively low, especially in the high suction range, the flow rate is extremely slow. As a result, the duration of such test is often quite long. The second difficulty is to maintain a good contact between the sample and the porous stones to ensure the continuity of the water flow. Indeed, the volume of the soil can decrease when applying a high suction [8], [9].

Therefore permeability measurement of unsaturated soils is a very time-consuming process. The duration of the test increases as the water content in the soil decreases. The permeability values can differ by several orders in magnitude causing direct measurement to be very difficult as there is no apparatus that can measure such a wide range of permeability values efficiently. Permeability measurements can be performed either in the field or in the laboratory. However, field measurements are usually more variable due partly to macroscopic features and partly from the assumptions made [9], [10].

In situ tests are not applicable when the horizontal flow is significant and for matrix suctions exceeding 50 kPa because the drainage phenomenon becomes extremely long. In addition, the boundary conditions are not well controlled in situ.

## **3. Critics of mathematical models**

Indirect permeability measurements are generally performed by establishing permeability functions and exploiting the relationship between water content and pore water pressure. Many functions are available in the literature, in general they can be classified into three groups: empirical, macroscopic and statistical [10].

The empirical equations are derived from the need for an equation to describe the variation in permeability as a function of matrix suction or water content. They are therefore simple mathematical relationships of purely experimental origin [11]. However, this approach is to be used with great precaution because none of the relationships is valid for all soils. Even if a formula proves to be adequate for a soil class, it must be recognized that the coefficients can vary considerably from one soil to another.

The objective of macroscopic models is to obtain an analytical expression for the permeability function. All macroscopic models show the analogy between laminar flows (microscopic scale) and flow in porous media (macroscopic scale). The flow is then solved for a simple laminar flow system, by correlating macroscopic variables such as mean flow velocity, hydraulic gradient and hydraulic radius [11]. The main criticism of macroscopic models is that these models neglect the effect of pore size distribution [6].

Statistical models are the most rigorous models for the representation of permeability functions [2]. For these models, the coefficient of permeability is derived from the soil-water characteristic curve [3]. Parameters of these models are often expressed as mean for different classes of textures with considerable uncertainty in the predicted  $k(\Theta)$  function,  $k(\Theta)$  being the permeability as a function of the volume of water content. The problem is aggravated when the number of parameters increases especially if the data  $h(\Theta)$  are also based on a model,  $h(\Theta)$  being the hydraulic gradient [12].

#### 4. Evaluation of the permeability of unsaturated soils

Comparisons between the results of the different models proposed in the literature often neglect case studies and results are often presented as percentage of errors between models and laboratory results. In the following, the results of three prediction models are examined:

##### **Brooks & Corey (model [2])**

$$K(\Psi) = k_s \begin{cases} \left(\frac{\Psi_b}{\Psi}\right)^{2 + \left(\frac{5\lambda}{2}\right)} & (\Psi > \Psi_b) \\ 1 & (\Psi \leq \Psi_b) \end{cases} \quad (1)$$

$K(\Psi)$ : Permeability.

$k_s$ : Permeability in the saturated state of the estimated soil.

$\Psi_b, \lambda$ : Adjustment parameters.

$\Psi$ : Suction

**Campbell Estimation [11]**

$$K(\Psi) = k_s \cdot \left(\frac{\theta_s}{\theta}\right)^{2+\frac{2}{b}} \quad (2)$$

$\Theta_s$ : Water content in the saturated state.

$\Theta$ : The water content at a given soil suction calculated from the water retention curve.

$b$ : Parameter used to vary Campbell's estimate.

**Kunze and al. model [11]**

$$k(\Psi_i) = \frac{k_s T_s^2 g \theta_s^p}{k_{sc} 2\mu_w n^2} \sum_{j=i}^m [(2j + 1 - 2i) \Psi_j^{-2}] \quad (3)$$

With:

$k(\Psi_i)$ : coefficient of permeability calculated for a specific water content,

$\Theta_i$ , corresponds to the  $i$ th interval.

$i$ : Number of intervals

$j$ : Number from  $i$  to  $m$ .

$k_{sc}$ : Calculated saturated permeability coefficient.

$T_s$ : Tension of the surface of the water.

$P_w$ : Density of the water.

$g$ : Gravitational acceleration.

$\mu_w$ : Viscosity of the water.

$P$ : Constant that takes into account the interaction of pores of different sizes.

$m$ : Total number of intervals

All the samples used in the rest of this article are published on the SoilVision database. Each set of samples have the same features (same type of soil, same origin, same percentage of sand, silt or clay, the same degree of saturation ....). The saturated and unsaturated permeabilities of these soils are measured in the laboratory according to different measurement methods following the drying process.

**4.1. Evaluation of the permeability of a set of sand by Brooks and Corey method**

Brooks and Corey method is applied to a set of sand. Saturated permeabilities  $k_s$  were estimated using 3 methods that are laboratory tests, air entry value and Terzaghi model. The results are represented in figure 1 below:

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Fig. 1. Errors presented by Brooks and Corey method for a set of sand as a function of ks models (laboratory ks, air entry value, Terzaghi)

From this graph we can clearly see that the Brooks and Corey method, designed to estimate the permeability of unsaturated sands, did not provide the same result with all methods of ks estimation.

*Special case 1:*

The characteristics of four samples of sand extracted from a site in Netherlands are shown in the table below:

Table 1

**Characteristics of sand samples extracted from a site in Netherlands**

Soil counter	Country	State	Region	Site	Saturation	e (void ratio)	W (water content)	% organic	% sand
11369	Netherlands	Kootwijk		A	76.89 %	0.85	24.76	0.27	98.73
11368	Netherlands	Kootwijk		A	76.89 %	0.85	24.76	0.27	98.73
11370	Netherlands	Kootwijk		A	80.37 %	0.85	25.87	0.27	98.73
11371	Netherlands	Kootwijk		A	89.59 %	0.70	23.88	0.27	98.73

Figure 2 below illustrates the evaluation of the unsaturated permeability of these four sand samples according to the method of Brooks and Corey:

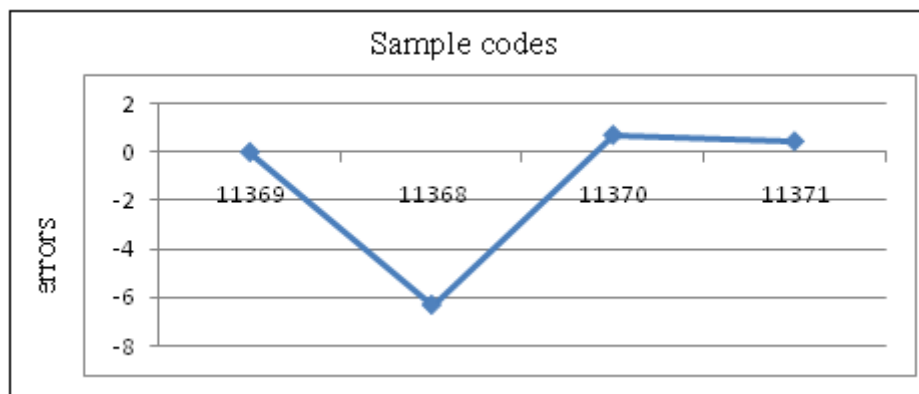


Fig. 2. Errors presented by the method of Brooks and Corey for a set of sand extracted from Netherlands with ks evaluated in the laboratory

We can observe that for the same type of soil (sand), the same sampling location, the same percentage of sand and organic matter, almost the same degree of saturation, the Brooks and Corey model did not show the same performance for these samples considered identical. This is because a small error in the estimation of the parameters of this model can induce a big error in the estimation of  $k_u$ .

#### 4.2. Evaluation of the permeability of a set of sand by the Kunze method

Let the same set of sand as that chosen to evaluate the method of Brooks and Corey. The errors of the estimate of  $k_u$  according to the Kunze method for different methods of evaluating  $k_s$  are presented in Figure 3 below;



Fig. 3. Errors presented by the method of Kunze for a set of sand as a function of  $k_s$  model (laboratory  $k_s$ , air entry value, Terzaghi)

From this graph we can clearly see that the Kunze method, which is classified among the statistical methods known as the most rigorous, did not provide the same result for all the samples.

#### Special case 2:

Details of two samples extracted from the SoilVision database are shown in the table below:

Table 2

Details of sand samples extracted from the SoilVision database

Soil counter	Country	State	Region	Site	Saturation	e (void ratio)	W (water content)	% organic	% sand
10929	USA	CA	Contra Costa County	D2F1aC1	97 %	0.58	20,72 %	0	90,97 %
10930	USA	CA	Contra Costa County	D2F1aC1	100 %	0.61	22,39 %	0	90.97 %

According to this table, the two soils are of the same type (sand) as they are taken from the same American site and they do not show a great difference in the degree of saturation or in the water content. They have the same percentage of sand and organic matter. Figure 4 illustrates the evaluation of  $k_u$  errors of these two sand samples made according to the method of Kunze.

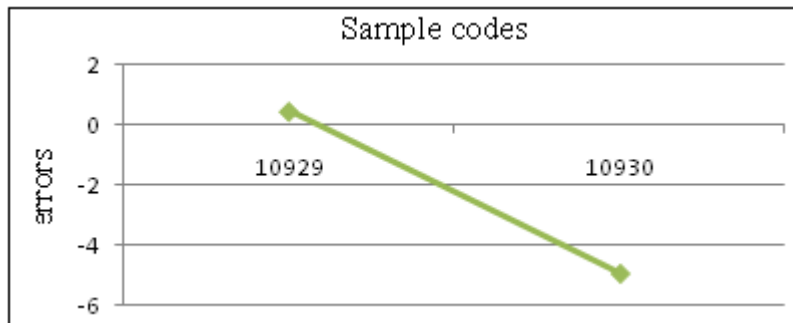


Fig. 4. Errors presented by the Kunze method for a set of sand extracted from USA site with  $k_s$  evaluated in the laboratory

For the same type of soil (sand), the same sampling location, the same percentage of sand and organic matter, the degree of saturation is very close for the four samples. The Kunze model did not show the same performance for these samples considered identical because a small error in the estimation of the parameters of this model can induce big error in the estimate of  $k_u$ .

#### 4.3. Evaluation of the permeability of a set of sand using the Campbel method

We will use again the sand to evaluate the method of Brooks and Corey. The errors in  $k_u$  estimation made according to the method of Campbel are shown in Figure 5 below:

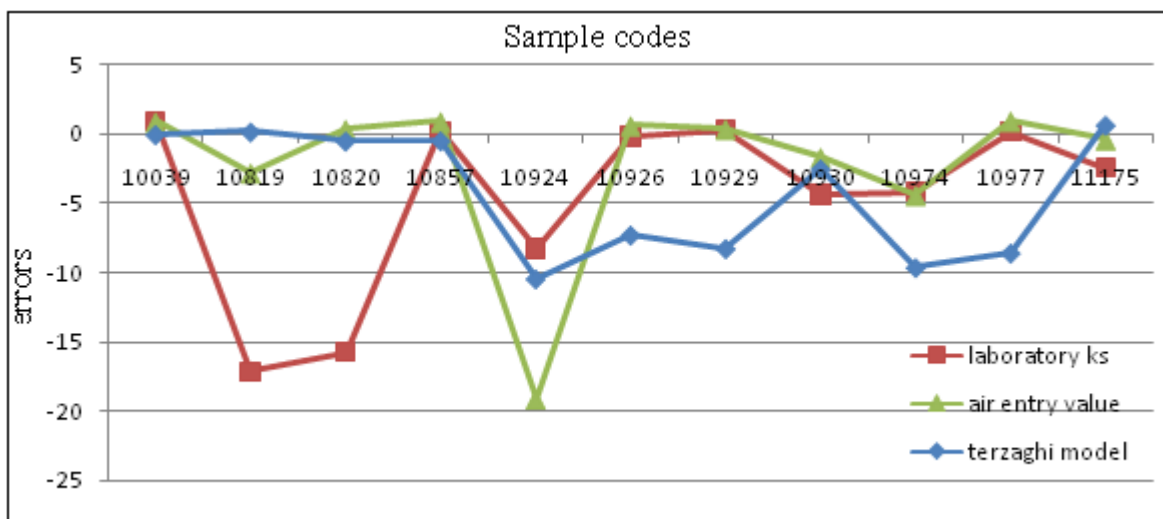


Fig. 5. Errors presented by the method of Campbel for a set of sand as a function of  $k_s$  models (laboratory, air entry value, Terzaghi)

From Figure 5 we clearly see that the Campbel method, which is classified as one of the most rigorous statistical method, did not show the same result for all samples.

#### 4.4. Comparison between the methods of Campbel, Kunze and Brooks and Corey

Figure 6 below presents the results obtained by the methods of Brooks and Corey, Kunze and Campbel for the same set of sand

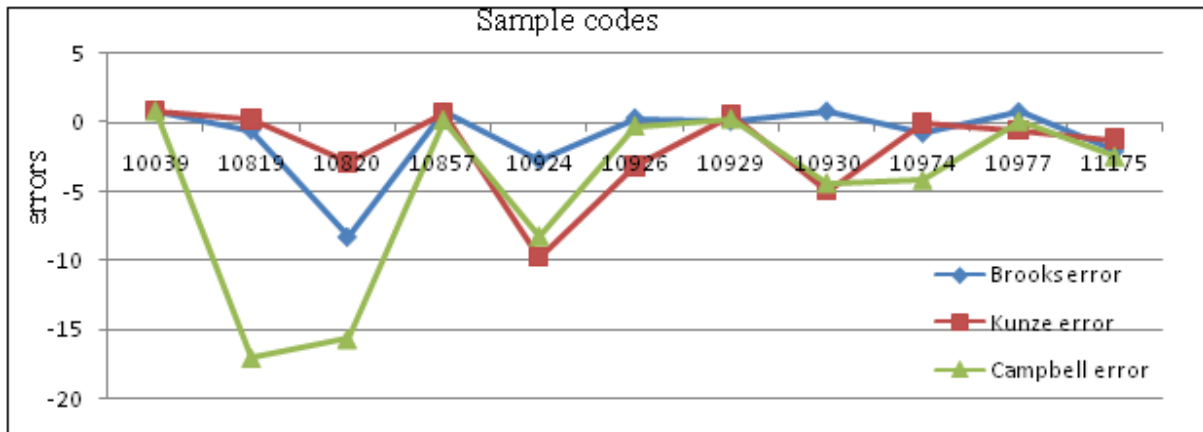


Fig. 6. Comparison between the results obtained by Brooks and Corey, Campbel and Kunze methods

From Figure 6 it can be observed that there is not a single sample that has shown poor results with all models, however, there are samples that have shown good results with all three models. So this is evidence that one must always look for the best model that presents convergent results to the laboratory data and not confide in the overall remarks that rank the models according to their percentages of good estimates for a soil type since it can lead to major errors in the design of civil engineering works.

#### 4.5. Evaluation of permeability of a set of silty clay by the Brooks and Corey Method

Figure 7 below shows a silty clay set with  $k_u$  estimated according to the Brooks and Corey method for  $k_s$  measured by 3 methods (laboratory, air entry value and Terzaghi);



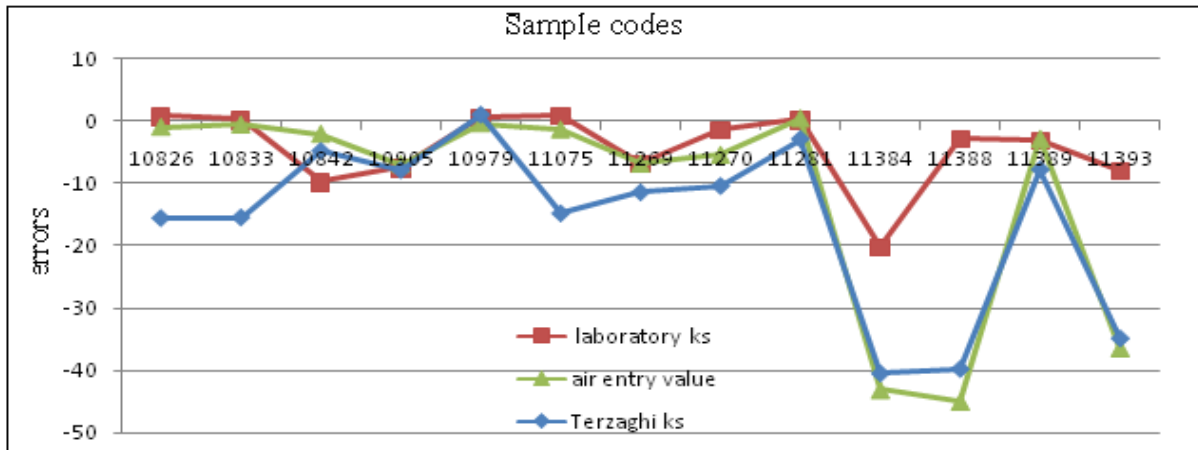


Fig. 7. Errors presented by the method of Brooks and Corey for a set of silty clay as a function of  $k_s$  models (laboratory, air entry value, Terzaghi)

From this graph we can clearly see that the method of Brooks and Corey did not present the same type of result for all the samples with different method of  $k_s$  estimation.

**Special case 1:**

The characteristics of two silty clay samples extracted from a site in Switzerland are given in the table below:

Table 3

**Details of Silty clay samples extracted from a site in Switzerland**

Soil counter	Country	State	Region	Site	Saturation	e (void ratio)	w (water content)	% organic	% silt	% clay
11114	Switzerland	Langenthal		Riedhof	45.90 %	10.76	189.3	0	62.38	31.16
11115	Switzerland	Langenthal		Riedhof	100 %	0.59	22.6	0.20	66.7	28.4

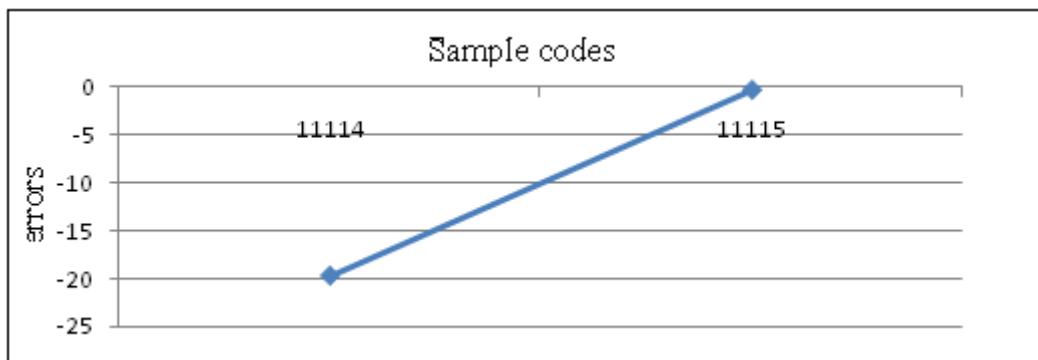


Fig. 8. Errors presented by the method of Brooks and Corey for a set of sand extracted from Switzerland with  $k_s$  evaluated in the laboratory

From Figure 8 we find that for the same type of silty clay soil, the same sampling location, the same percentage of sand and organic matter, the degree of saturation is very close for both samples (11114, 11115). The Brooks and Corey ku estimation model did not show the same performance for the two samples considered identical. It has presented a poor estimate for the first sample and a good estimate for the second.

#### 4.6. Evaluation of the permeability of a set of silty clay samples by the Kunze method

The errors of ku estimation of a set of silty clay by Kunze method are shown in Figure 9 below:

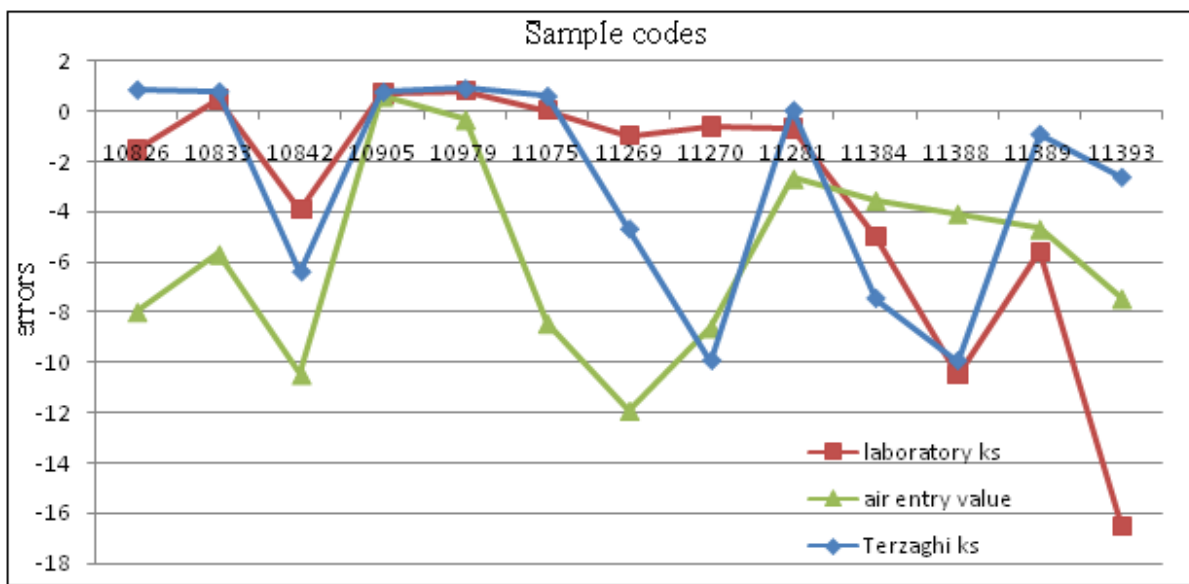


Fig. 9. Errors in ku presented by Kunze method for a set of silty clay with ks evaluated by 3 methods (laboratory, air entry value, Terzaghi)

From this graph we can clearly see that the Kunze method, which is classified among the statistical methods known as the most rigorous did not provide the same result for all the samples.

#### 4.7. Evaluation of the permeability of a set of silty clay using the Campbel method

Figure 10 shows results of errors in ku for the set of silty clay used previously

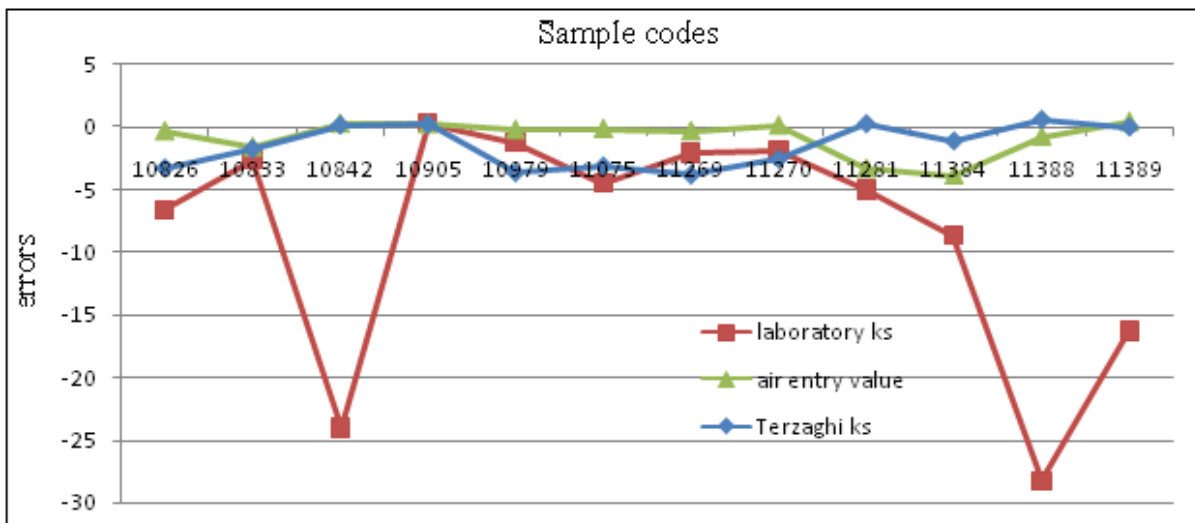


Fig. 10. Errors in  $k_u$  presented by the Campbell method for a set of silty clay with  $k_s$  evaluated by 3 methods (laboratory, air entry value, Terzaghi)

As with the Kunze method, we can clearly see that the Campbell method, classified among the statistical methods known as the most rigorous, did not show the same type of result for all the samples.

## 5. Discussion of results

Comparing the results presented by the Campbell method for sand or silty clay associated with  $k_s$  evaluated in the laboratory and Terzaghi method we noticed that the second case showed better results than the first case. This result is unexpected because:

- firstly, it is a fine soil and the Terzaghi method is designed primarily for sandy soils so the good result in this case gives us pause for thought.
- Better results were expected with the most credible laboratory data.

This confirms once more that there is no general rule that allows the selection of a model  $k_s$  associated with a model  $k_u$  to ensure good prediction of  $k_u$  of a given soil.

## 6. Conclusions

Models for predicting the coefficient of permeability of unsaturated soils are numerous and the choice between these models to evaluate  $k_u$  without the risk of a large error is difficult. Therefore some researchers have classified these models under three groups (macroscopic, empirical and statistical).

To shed light on the difficulty in choosing a model that will provide a good prediction of  $k_u$ , we have chosen two different soil textures; coarse soil (sand) and fine soil (silty clay). Statistical and empirical models were also chosen to evaluate the unsaturated permeability of these two types of soils. We assessed the saturated permeability of the

two sets of soil samples by the three  $k_u$  estimation models based on some  $k_s$  estimation models (laboratory data, air inlet pressure and Terzaghi).

The results of this study show that all the models can present very good results for some samples and very bad results for others and this for the case of the two textures of soils. The same remark was made for the samples extracted from the same site which have identical characteristics. The same applies to the results of these models associated with different methods for estimating the permeability of soils in the saturated state.

We have also noticed that a model designed for the prediction of one type of soil will be able to present very good results for another type of soil. The case of Brooks and Corey model designed for the prediction of the  $k_u$  of sands, presented very good estimates for silty clay samples.

However, it is clear that there is no one model that shows good results for one type of soil or poor results for another type of soil. So, there is not a general rule that allows one to select one model among others that will be able to predict the permeability of an unsaturated soil without the risk of a large error.

To remedy this problem, we recommend going through a series of comparisons between the results of several models and then choosing the one that presents the best result. As this process consumes a lot of time we propose to develop a help system that will be able to evaluate the permeability according to several models, compare between their results and choose the most efficient in just a few clicks.

## References

- [1] B. Wagner, V. R. Tarnawski, V. Hennings, U. Mueller, G. Wessolek, R. Plagge, "Evaluation of pedo-transfer functions for unsaturated soil hydraulic conductivity using an independent data set", *Geoderma Elsevier*, 2001.
- [2] A. Teamrat, J. Timothy, W. Grace, "Correspondence of the Gardner and van Genuchten–Mualem relative permeability function", *Water Resources Research*, vol. 43[15], 2007
- [3] SoilVision. "A Knowledge-Based Database System for Soil Properties", SoilVision Systems Ltd., Saskatoon, Saskatchewan, Canada, 2001
- [4] R. J. Kunz, G. Uehara and K. Graham, "Factors important in the calculation of hydraulic conductivity", *Proc., Soil Sci. Soc. of Am.*, 32, 1968, pp. 760-765
- [5] G. S. Campbell, "A simple method for determining unsaturated conductivity from moisture retention data", *Soil Sci.* 117, 1974, pp. 311-314
- [6] R. H. Brooks and A. T. Corey, "Hydraulic properties of porous media", *Hydrology papers*, 1964
- [7] L. Mechkarini, A. Bali, F. Kaoua, "Système d'aide pour l'évaluation du modèle de l'estimation de la perméabilité non saturée de Leong et Rahardjo (1997)", *Colloque International Caractérisation et Modélisation des Matériaux et Structures*, CMMS14, Tizi Ouzou Algérie, 2014
- [8] D. G. Fredlund, A. Xing, and S. Huang, "Predicting the permeability function for unsaturated soil using the soil water characteristic curve", *Canadian Geotechnical Journal*, Vol.3, 1994, pp. 533-546
- [9] M. H. Jokar, A. Khosrovi, A. Heidaripناه and F. Soltani, "Unsaturated soils permeability

- estimation by adaptive neuro-fuzzy inference system”, *Soft Computing*, Springer, 2018
- [10] H. Ram, C. YiFeng, L. HuiHai and Z. Chuang Bing, “A relative permeability model for deformable soils and its impact on coupled unsaturated flow and elasto-plastic deformation process”, *Science China Technological Sciences*, Vol 58 No 1, 2015, pp. 1971-1982.
- [11] D. K. Todd and L. W. Mays, “*Groundwater Hydrology*”, John Wiley & Sons, New York, 2005
- [12] D. Chacaborty, A. Chacaborty, P. Santra, R. K. Tomar, R. N. Garg,,R. N. Sahoo, S. G. Choudhury, M. Bhavanarayana and N. Kalra, ”Prediction of hydraulic conductivity of soils from particle-size distribution”, *Current Science*, Vol. 90, No. 11, 2006, pp. 1527-1531