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Modeling the Drying Shrinkage of Structural Concretes

Modelarea contracției prin uscare a betoanelor structurale

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Abstract. *Shrinkage in hydraulic materials is a complex time-dependent process. For standard concretes, one of the most significant parts of shrinkage is drying shrinkage. In fact, to predict deformations of concrete due to shrinkage, various predictive models have been developed; most of them use many numbers of factors that can affect shrinkage such as concrete strength, concrete age at loading, curing conditions type, ambient conditions, type of cement and aggregates, water to cement ratio, concrete mix, member shape and size, loading duration and type. A such number of parameters increases the complexity of using these models lead to some prediction imperfections; thence a new simplified model is needed. The main target of the current paper is to formulate a novel and simplified model with a minimum of factors that affect drying shrinkage behaviour as like as relative humidity and volume to area ratio of the concrete element (V/S). To achieve this goal, we had developed a prediction model based on probability density function and a small number of parameters that influence shrinkage like relative humidity and volume to surface ratio of the element. A huge database has been used to calibrate our model's parameters using the most recent studies and researches to validate the model.*

Key words: hydraulic materials, drying shrinkage, modelling, prediction, concrete deformation, structural concrete.

1. Introduction

To predict concrete shrinkage behavior, diverse analytical models have been elaborated and some of them are approved by diverse codes and recommended by famous researchers [1].

Shrinkage is affected by multiple variables as well as concrete strength, concrete loading time, cement type, type of curing conditions, ambient conditions, water to cement ratio, concrete mix, member size and shape, aggregates type, duration and type of loading, etc.[2]. This large number of parameters affecting shrinkage increases the complexity of utilizing these models and can lead to imperfections in shrinkage predictions. It can also reduce databases exploitable results due to the lack of one or more of these parameters. Thence, a new simplified prediction model containing fewer parameters that affect shrinkage phenomena is necessary. The quality of a shrinkage predictive model depends on the contribution of each parameter which conducts the phenomena [3]. In its report 209.1R-2 [4], the American Concrete Institute (ACI) defines shrinking as the deformation measured on a load-free concrete sample. ACI states that shrinkage excludes changes in length due to variations in temperature, but it depends on the environment, configuration, and size of the specimen.

Researchers must often describe and analyze phenomena in diverse areas of research, with actions understood only from laboratory observations. For this reason, the synthesis of a mathematical model with similar behavior to the actual phenomenon is of interest. In particular cases, with the understanding of the model parameters and the experience requirement of the phenomenon, we are able to suggest a mathematical model named a deterministic model. The exact mechanisms of the phenomenon, though, are generally unknown. We might, therefore, formulate a mathematical model on which we determine the parameters of measurements from samples.

The drying shrinkage in concretes is the most significant part of shrinkage deformations [5]; it results from the reduction of pores' relative humidity which increases directly the capillary tension of pores occupied the water and in the solid surface tension at pore walls. The data from experimental results show that the measured ultimate values of concrete drying shrinkage of many specimens had a nonlinear function of the ambient relative humidity [6].

This study aims to develop a representative prediction model of drying shrinkage of hydraulic materials with fewer affecting factors and more predictive accuracy. This manuscript is structured in the following way: First, we expose the relation between drying shrinkage development and the probability density function. In Section 2, 3, and 4, we present step by step the demonstration that leads from mathematical density function to a model that can describe the evolution of drying shrinkage; these model parameters were estimated by using large experimental results gathered in different databases then simplified to reduce the number of parameters. In Section 5, we present a comparison of our developed model and various American and European prediction models that existed in the literature. Finally, section 6 summarizes the main conclusions.

2. Experimental Investigation

The present research is based on a vast range of experimental results obtained in various American and European laboratories by internationally renowned researchers [7] and [8].

The experimental results analyzed are those that include the most parameters influencing the drying shrinkage.

3. Analytical Investigation

The current study is founded on a statistical study of experimental results given and summarized by Bazant [8]. The analysis of these results presents the dominant values between [300-600 µm/m] that most frequently present shrinkage deformation value.

4. Modeling

In our case, the drying shrinkage evolution is described by a curvature that starts by exponential shape thence it advances towards an asymptotic limit, as shown in Fig.1. In statistics, this form of shapes matches whit the curve of the density probability function F (t, t_0) obtained with direct integration of the density probability function F (t, t_0) in function of time (t), as seen in Fig. 2.

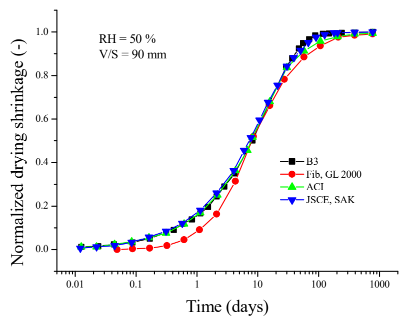


Fig. 1. Normalized drying shrinkage evolution [9].

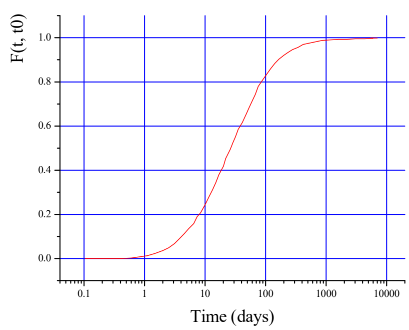


Fig. 2. Probability density function F (t, t0) [7].

For this case of study, the probability density function *f (t, t_0)* given in Eq.(1) is two parameters Weibull function [10]:

$$f(t,t_0) = \frac{c}{t-t_0} \times \left(\frac{t-t_0}{t-t_0}\right)^{C-1} \times \exp\left\{\left(\frac{t-t_0}{t-t_0}\right)^{C}\right\} \tag{1}$$

Such that $t\text{-}t_0 > 0$,
where t_0 is the time of loading, and $f(t, t_0)$ is a probability density function.

For the resolution of this equation, let:

$$b = (t - t_0)^{-C} \Rightarrow \quad b = \frac{1}{(t - t_0)^C}$$

and $\quad y = t - t_0$

Hence, Eq. (1) becomes as follows:

$$f(t, t_0) = \frac{c}{(t - t_0)} \times \frac{(t - t_0)^{C-1}}{(t - t_0)^{C-1}} \times \exp\left\{\frac{(t - t_0)^C}{(t - t_0)^C}\right\} \Rightarrow f(t, t_0) = c \times \frac{(t - t_0)}{(t - t_0)^C} \times \exp\left\{\frac{(t - t_0)^C}{(t - t_0)^C}\right\} \quad (2)$$

Therefore;

$$f(t, t_0) = c \times b \times y^{C-1} \times \exp\left(- b \times y^C\right) \quad (3)$$

The probability density function is given by: $\quad F(t, t_0) = \int f(t, t_0)dt \quad (4)$

Proceed to the development of Eq. (4)

Such as $\quad F(t, t_0) = \int_{-\infty}^{0} f(t, t_0)dt + \int_{0}^{t} f(t, t_0)dt \quad (5)$

$$\int_{-\infty}^{0} f(t, t_0)dt = 0 \quad (6)$$

Replace the function $f(t, t_0)$ by the equation in the integral as follows:

$$F(t, t_0) = \int_0^t \begin{pmatrix} c \times b \times (t - t_0)^{(c-1)} \times \\ \exp(-b \times (t - t_0)^c) \end{pmatrix} .dt \quad (7)$$

Let:

$$v = b \times (t - t_0)^c \quad \Rightarrow \quad v' = c \times b \times (t - t_0)^{c-1} \quad (8)$$

by replacing v and v' in Eq. (5), we obtain

$\int v' \times e^{-v} = -e^{-v}$

Where:

$$F(t, t_0) = -e^{-b(t-t_0)^C}\Big|_0^t \quad = \quad 1 - e^{-b(t-t_0)^C} \quad (9)$$

To consider the development of the density probability function $F(t, t_0)$ to reach an asymptotic limit, we multiply the Eq. (9) by a non-zero positive number "a " which yields to the final form:

$$F(t,t_0) = a \times (1 - e^{-b(t-t_0)^c}) \tag{10}$$

In our case, the function *F (t, t₀)* represents the degree of progress of drying shrinkage *φ (t, t₀)* where:

$$\varphi(t,t_0) = a \times (1 - e^{-b(t-t_0)^c}) \tag{11}$$

4.1 Estimation of the Model Parameters

To identify the model parameters, we used the results of the tests given by Bazant [7]. These test series involve 35 cylindrical samples of diameter 160 mm and 36 cylindrical samples of diameter 83 mm. also besides; three cylindrical samples of 300 mm diameter are also measured. The length of all cylinders is double their diameter.

The most appropriate and simplest method for estimating the parameters of linear models is the least-squares method [11]. This method consists of minimizing the differences between the regression line and the explained variable "*y*"; in other words, it reduces the sum of the squares also called the "sum of the squares of the residues" denoted "*SCR*".

$$SCR = \sum_{i=1}^{N} \varepsilon_i^2 \tag{12}$$

With, $\varepsilon_i = y_i - \hat{y}_i$: error at the point *t* between the measured and calculated value.
The $\hat{\beta}$ estimation is the value of β which renders the expression (12) minimal

$$y_i = \hat{\beta}_0 + \hat{\beta}_1 x_{1i} + \hat{\beta}_2 x_{2i} + \ldots\ldots.. + \hat{\beta}_k x_{ki} + \varepsilon_i \tag{13}$$

The matrix form of this expression is:

$$y_N = X_{Nk} . \hat{\beta}_k + \varepsilon_N \tag{14}$$

The system (14) resolution allows the determination of the $\hat{\beta}$ estimator.

$$\hat{\beta} = (X'.X)^{-1} X'.Y \tag{15}$$

The degree of validity of a regression model is based on the following conditions [11]:
-The $\bar{R}^2$ must be as high as possible.
-Student's and Fisher's tests must provide acceptable results.
- The standard deviations of the coefficients must be the lowest on the estimated values of the coefficients.

From the set of observations on the variables of the model selected during our study, we have proposed several expressions by multiple regressions giving the parameters of the model Eq. (11); the expressions retained are given by the relations (16), (17), and (18).
With: *V/S*=volume area ratio, in (mm), and *RH*=relative humidity in (%).

The test's parameters of model coefficients *a, b* and *c* are given in Table 1, Table 2, and Table 3.

$$a = \beta_1 + \beta_2.(RH)^2 + \beta_3\left(\frac{V/S}{RH}\right) \tag{16}$$

$$b = \beta_4 + \beta_5.(V/S) + \frac{\beta_6}{RH} \tag{17}$$

$$c = \beta_7 + \beta_8\left((V/S)^2.(RH)\right) \tag{18}$$

Model Coefficients		Standard Deviation	Student's test		Fisher's test		Correlation coefficient	
			$T_{Student}$	*P (S)**	T_{Fisher}	*P (F)***	R^2	$\overline{R}^2$
β_1	1.25004	0.0155	80.640	0.00	682.51	0.0000	0.9572	0.9558
β_2	-0.8423	0.0267	-30.82	0.00	682.51	0.0000	0.9572	0.9558
β_3	-0.0012	0.0001	-9.607	0.00	682.51	0.0000	0.9572	0.9558

Table 1.

Test « a » parameter's tests

Model Coefficients		standard deviation	Student's test		Fisher's test		Correlation coefficient	
			$T_{Student}$	*P (S)**	T_{Fisher}	*P (F)***	R^2	$\overline{R}^2$
β_4	0.236297	0.0027	85.8150	0.00	1071.38	0.0000	0.9723	0.9714
β_5	-0.00400	8.7E-05	-45.890	0.00	1071.38	0.0000	0.9723	0.9714
β_6	0.002927	0.0004	6.0662	0.00	1071.38	0.0000	0.9723	0.9714

Table 2.

Test « b » parameter's tests

Model Coefficients		standard deviation	Student's test		Fisher's test		Correlation coefficient	
			$T_{Student}$	*P (S)**	T_{Fisher}	*P (F)***	R^2	$\overline{R}^2$
β_7	0.540896	0.0037	144.6832	0.0000	682.517	0.0000	0.7901	0.7202

β_8	3.13E-05	8.9E-06	3.5082	0.0127	682.517	0.0000	0.7901	0.7202

Table 3.

Test « c » parameter's tests

P (S)* : Probability of significance of each estimated coefficient.
P (F)**: Probability of significance associated with T_{Fisher} value.

4.2 Improvement of the Model

4.2.1 Adjusting the Parameter « *a* »

The parameter "a" represents the limit value of drying shrinkage; this parameter « *a* » is influenced by the relative humidity conditions *RH* and the *V/S* ratio of the element [3]. The relative humidity is one among the most essential factors affecting the final shrinking of concrete.

Fig. 3 [3] shows a reduction of the shrinkage for 14 and 28 days when the relative humidity tends to increase. By comparison, the shrinkage initially increases significantly for higher ages and then decreases.

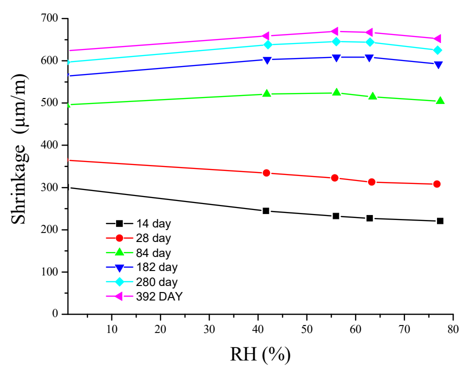


Fig. 3. Relative humidity *RH* effect on the shrinkage of concrete at different ages [3].

Figure 4 discusses the influence of the V / S ratio on the shrinkage measured for various concrete ages. The higher the V / S ratio, the lower the shrinkage.

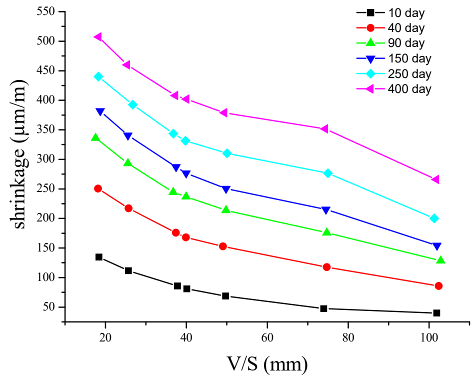


Fig. 4. *V/S* ratio effect on the shrinkage of concrete at different ages [3].

A statistical analysis founded on experimental results given by [8] shows that the values of the parameter "*a*" are mostly between [300-600 µm/m] as illustrated in Fig. 5.

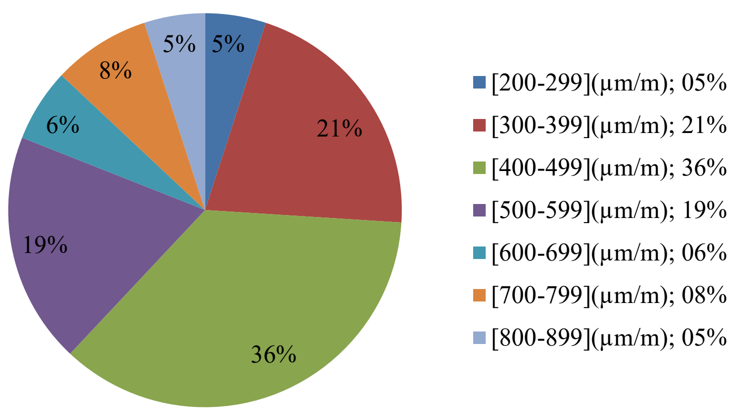


Fig. 5. Proposal of the values by interval of the parameter (a) (µm/m).

4.2.2 Adjusting the Parameter « *c* »

The variation of the parameter « *c* » is a function of the relative humidity *RH* and *V/S* ratio as reported in Table 4.

Table 4.

« c » coefficient values predicted in our model

V/S	RH	c	V/S	RH	c	V/S	RH	c
76 mm	20%	0.59	102 mm	20%	0.536	152 mm	20%	0.599
76 mm	50%	0.44	102 mm	50%	0.508	152 mm	50%	0.576
76 mm	75%	0.51	102 mm	75%	0.525	152 mm	75%	0.653

Note that the "c" parameter values vary little and is close to 0.5.
Adopting $c = 0.5$, the equation yielding the progression of the drying shrinkage is reduced to an expression with two parameters:

$$\varphi(t,t_0) = a \times (1 - e^{-b(t-t_0)^{0.5}}) \tag{19}$$

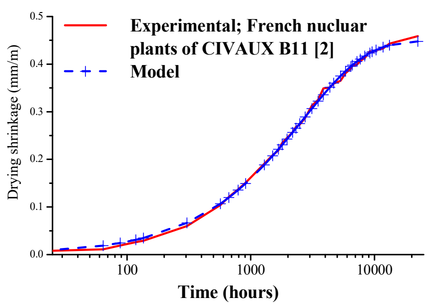
5. Validation of the Model

To validate our model, we first compare the model predictions with experimental results for high-performance and normal concrete given by [12] and [13] and then with most common American and European models such as the ACI, Sakata, B3, B4, B4S, CEB 99, GL2000, Idiart, SANS 10100, Wits, Fib and EC2 model.

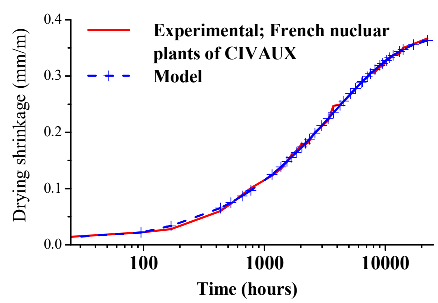
5.1 Confronting Drying Shrinkage Model Evolution of High-Performance Concrete With Experimental Results of Granger [12].

The high-performance concrete constitutes a new concrete with novel characteristics. It is useful to compare the developed model prediction with the experimental results obtained by Granger [12].

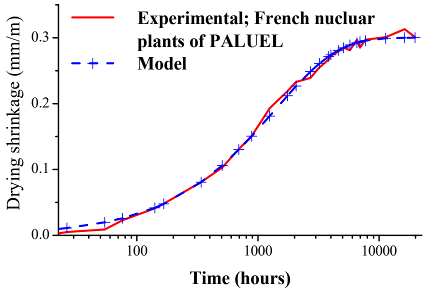
The results are grouped by model and experimental results in Fig. 6 (experimental results obtained by Granger [12] for French nuclear plants of CIVAUX, PALUEL, and CHOOZ). Our predictive model tends to align well with the development of high-performance concrete drying shrinkage.



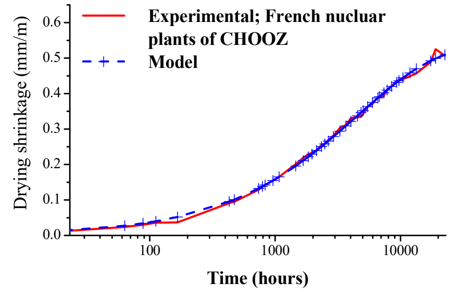
6.a.



6.b.



6.c.

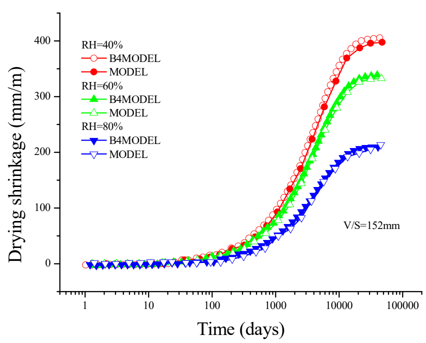


6.d.

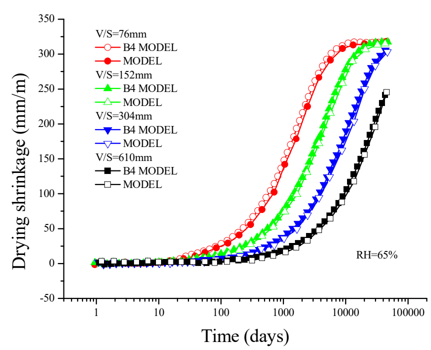
Fig. 6. Confronting drying shrinkage model evolution of high-performance concrete with experimental values obtained by [12] for French nuclear plants of CIVAUX, PALUEL, and CHOOZ**.**

5.2 Comparison of Model Predictions With Experimental Values of Bazant and al [13]

Figure 7.(a) shows the contribution of the drying shrinkage for a constant volume to surface ratio *V/S*=152 mm and variable environmental relative humidity *RH* (40%, 60%, 80%), as well as constant environmental relative humidity of *RH*=65% and variable *V/S* ratio (76; 152; 304; 610 mm) in Fig. 7.(b).



7.a.



7.b.

Fig. 7. Predictions of drying shrinkage curves as a function of varying *V/S* ratio and humidity, and B4 model [13].

In the curves of Fig. 7, we provide a comparison between our model prediction and normal concrete drying shrinkage with different *RH* and *V/S* ratio.

Curves in Fig. 7.(a) shows the influence of relative humidity (*RH*) for constant *V/S* ratio. This influence is less nuanced at younger ages (the first days), but it clearly appears at advanced ages. We observe on the curves of Fig. 7.(a) that the amplitude of the drying shrinkage increases inversely with the decrease of the relative humidity *RH*.

The curves in Fig. 7.(b) illustrate the influence of the size of concrete element (*V/S* ratio). It is clearly appears that the effect of the dimension of the elements (*V/S* ratio) is more marked. We observe on these curves that the drying shrinkage decreases with increasing *V/S* ratio of specimens.

5.3 Confrontation of Developed Model Predictions With Most Common Models, Recent Research and Various Databases

5.3.1 Confrontation With Goel and al [2]

Figure 8 presents the prediction of drying shrinkage deformations obtained from six analytical models: ACI 209 model[4], the B3 model [9], the CEB-FIP model [14], GL2000 model [15], Muller model [16], and our prediction model.

To compare these varied prediction models, results predicted by these models are to be compared with experimental results. therefore, the experimental results of Russell and Larson [17] and the RILEM data bank [9] have been selected.

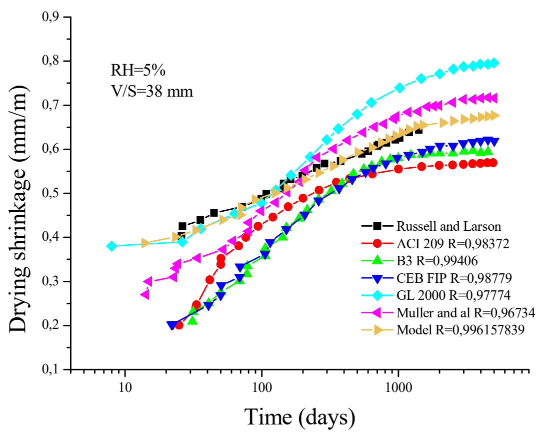


Fig. 8. Drying shrinkage predictions in concrete with V/S=38 mm and RH=5%.

From Fig. 8, in the GL2000 model it is observed an underestimation of experimental concrete shrinkage up to around 160 days of age and overestimates it after this age. Also, the Muller model presents an overestimation of experimental shrinkage after about 180 days. The B3 model and ACI model underestimates the experimental results. Our prediction model underestimates experimental shrinkage to the age of about 100 days then fit perfectly with it, these underestimations may be caused by other factors influencing shrinkage desiccation such as the concrete age at loading, cement type, aggregates, water-cement ratio, concrete mix, loading type, loading duration, etc.

5.3.2 Confrontation With Idiart and al [18]

In the curves of Fig. 9, we provide a comparison between predictions of the developed model and three simulation models (parabolic, linear and constant shrinkage coefficient) given by [18].

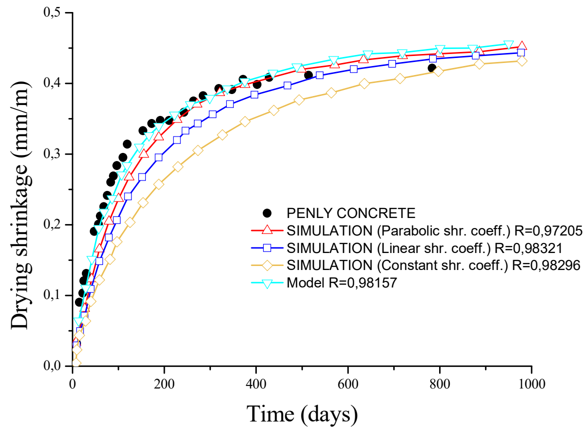


Fig. 9. Comparison of the developed and Idiart [18].

From Fig. 9, it is observed that all three simulations have the same beginning and ending rate of drying shrinkage but linear and constant shr coef underestimate experimental results at middle ages. Parabolic shr coef and our model fit perfectly and predict well the evolution of drying shrinkage of PENLY concrete.

5.3.3 Confrontation With Gaylard and al [19]

The most detailed model comparisons in the published literature are focused on the RILEM database, a compilation of 490 concrete shrinkage profiles primarily from study groups in North America and Europe [8].

In Fig. 10, we present concrete drying shrinkage predictions for structural use on the basis of historical data for South African concrete shrinkage [19].

We used eight published models as comparisons with our model developed in this study: ACI 209 [4], RILEM B3 model [9], CEB MC90-99 [14], GL2000 model [15], SANS 10100 model [20], Eurocode 2 [21], Fib model [22], WITS model [19]

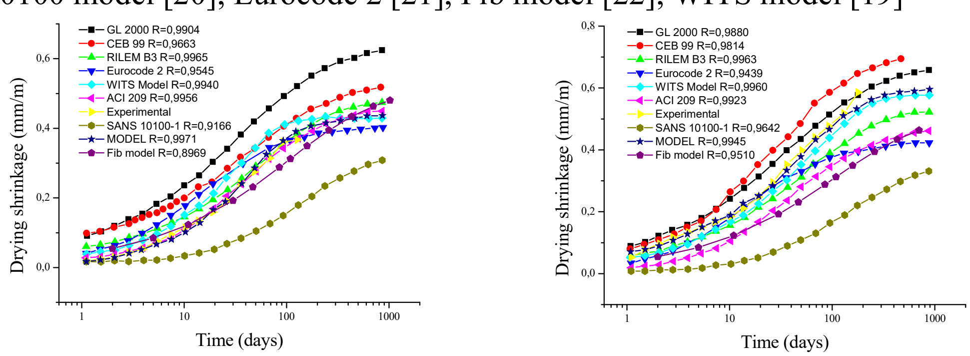
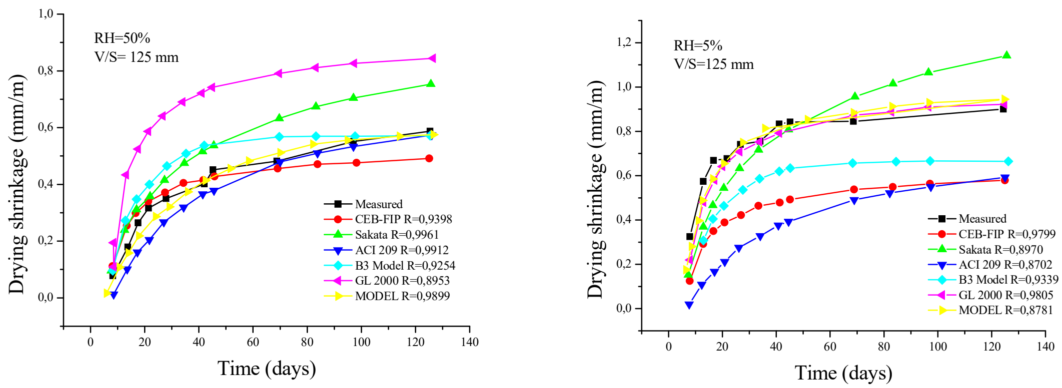


Fig. 10. Eight models predictions and experimental results [19] compared to our model.

It is obvious from Fig. 10 that the model developed was performing well, as expected. The models SANS 10100-1 [20] and GL2000 [12] appeared to underpredict and overpredict, respectively. No specific tendency concerning the performance of the other models is directly apparent from early and advanced age examination of the results.

5.3.4 Confrontation With Al-Saleh [1]

In Figure 11 we provide a comparison of experimental results measurements of drying shrinkage and theoretical drying shrinkage predictions applying the next five models: ACI 209 [4], CEB-FIP [14], B3 [9], Sakata [23], and GL2000 [15]. Measures on samples were taken with *V/S*=125 mm and various relative humidities *RH*=50% and *RH*=5%.



11.a. 11.b.

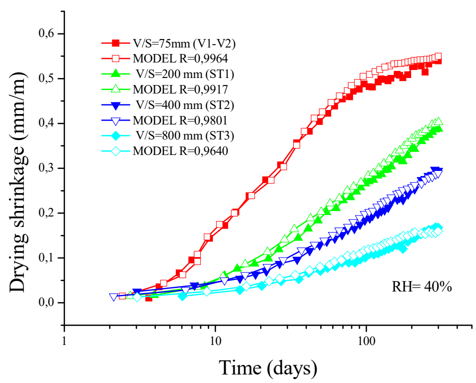
Fig. 11. Models prediction and drying shrinkage experimental measures [1].

Figure 11.(a) illustrates that ACI 209 and B3 models are the nearest theoretical drying shrinkage in *RH*=50%, particularly when approaching ultimate drying shrinkage. At the start of the drying-time, it is found that the ACI 209 model under predicts experimental drying shrinkage whilst the B3 model slightly over predicts experimental results. It can be noted from Fig. 11.(a) that CEB-FIP model at the beginning of the drying period is closed to the experimental drying shrinkage values. However, the model's predicted deformations start to deviate as time goes towards ultimate shrinkage from experimental drying shrinkage strains. The expected drying shrinkage using Sakata model is close to the experimental deformations after 40 days of drying time, subsequently, the values obtained increased with a fast rate as the drying time goes towards the ultimate. It is noticed that GL 2000 model has the worst prediction from early age until the end of the drying period. Our model predicts well the development of drying shrinkage compared to the results of experiments.

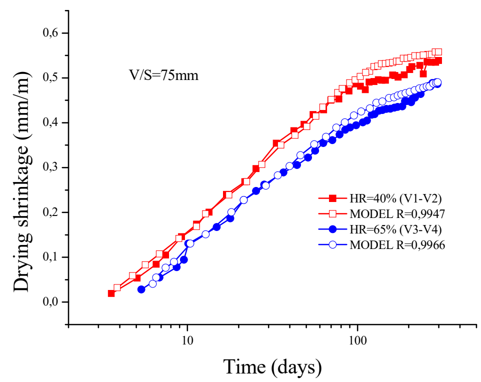
In Figure 11.(b) we observe an augmentation of the final experimental drying shrinkage rate due to very low relative humidity *RH*=5%. Our model and GL 2000 model describe well the evolution of experimental drying shrinkage. A rather large difference is observed at early and later ages for the rest of the models compared to experimental results. These imperfections are due to the presence of several parameters in these models.

5.3.5 Confrontation With Vinkler [24]

Three experimental specimens of *V/S*=200 mm (mentioned as ST1), *V/S*=400 mm (mentioned as ST2) and *V/S*=800 mm (mentioned as ST3) and standard cylinders of *V/S*=75mm were used. Cylinders were separated into two sets of two samples and retained under diverse environmental conditions: the initial group (mentioned as V1-V2) was maintained with the same ambient conditions as the specimens with 40% relative humidity, second group (mentioned as V3-V4) was maintained in controlled conditions with 65 % relative humidity. Different curing conditions allow for the determination of drying shrinkage and for comparison of drying shrinkage measured in cylinders (V1-V2, V3-V4) and in large concrete elements (ST1, ST2, and ST3).



12.a



12.b

Fig. 12. Predictions of drying shrinkage curves as a function of varying V/S ratio and humidity.

The thicker specimen shrinks less as intended due to the slower drying process. The strains in (ST1, ST2 and ST3) are smaller compared with the shrinkage of cylinders.

The far more interesting remarks from previous experimental data and model predictions are as follows:

1. The size impact has been illustrated quite clearly in Fig. 12.(a). The higher is volume-surface ratio, a slower shrinkage deformation was observed. The cause is obvious: thicker elements dry more slowly because the moisture moves and travels over a longer distance in the element and so the drying shrinkage acts consistently. This result is in correlation with [9] and [25].

2. Relative humidity variation affects directly the rate of drying shrinkage. We observe that the shrinkage rate increase with lower relative humidity condition as it is shown in Fig. 12.(b).

In Figure 13 we present a comparison to predict drying shrinkage of concrete and the most common models which are: Fib model [22], EC 2 [21], B4, B4S [13], and ACI 209 [4].

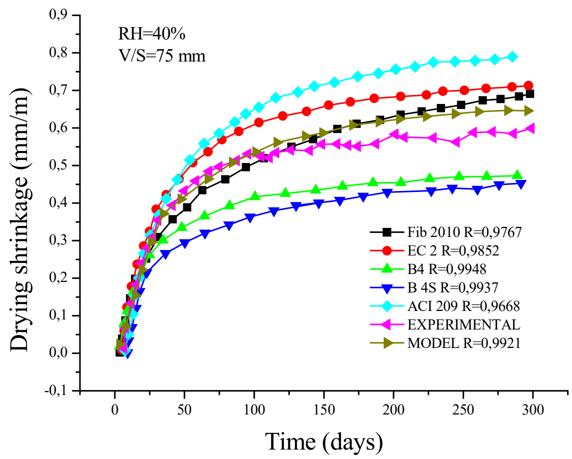


Fig. 13. Different models predictions with RH=40% and V/S=75 mm and [24] experimental results.

From Fig. 13, we note that drying shrinkage shows that the models exhibit a fast increase of strains at early age while the measured strain development is slower.

After a while, the rate of strains in forecasting models is reduced. ACI and EC2 models overestimate the evolution of drying shrinkage while B4 and B4S models underestimate it. Our model and Fib's model fit perfectly with experimental measurements at early ages and overestimate the evolution at later ages.

The analysis of these curves clearly shows the concordance behaviour that exists between the experimental results (of different researchers in different laboratories) and those predicted by our model. However, a slight difference is observed only on a few curves at the early age, and sometimes at later ages compared to experimental results.

The developed model well describes the evolution of concrete's drying shrinkage. Additionally, it presents better precision compared with the most common models.

6. Conclusion

The principal objective of this research was to develop a simplified predictive model with fewer affecting factors for drying shrinkage of structural concretes, namely for normal and high-performance concrete in order to predict the deformation rate during hardening. The main variables in the model are the volume-area ratio (V/S) and the relative humidity (RH). To reach this objective, we had based on a large number of experimental results obtained from various American and European laboratories and some current and important codes of practice. This step leads us to summarize four essential points:

1. The developed model is well adapted to represent and describe the evolution of drying shrinkage of high-performance concrete, and it has been validated by comparison with real experimental results.

2. The final developed model is very simple and easy to use and it presents the advantage of containing only two parameters in comparison with the necessary parameters of the other models.

3. The developed model can easily describe deferred deformations of concrete structures with more precision since the shrinkage strongly depends on desiccation.

4. It is a general model that applies particularly well to the range of conventional concretes such as ordinary concretes and high performance concretes and to concretes with similar characteristics to those of the latter. Extreme cases require the introduction of corrective factors.

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