

Study on the parameters influencing the bending and shear behavior of steel fiber reinforced concrete elements

Studiu privind parametrii care influențează comportarea la solicitarea de încovoiere cu forță tăietoare a elementelor din beton armate dispers

Muheeb Altaieb¹, Andrei Gîrboveanu¹, Dan Georgescu¹, Amine Rahaoui²

¹Technical University of Civil Engineering of Bucharest.
121-126 Bvd Lacul Tei, Bucharest, Sector 2, Romania
E-mail: andrei-sorin.girboveanu@phd.utcb.ro

²Construcții Erbașu SA.
72 Str Nicolae G. Caramfil, Bucharest, Sector 1, Romania

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Abstract. *Different types of fibers can be used to improve the mechanical behavior of steel fiber reinforced concrete. However, the degree to which this occurs also depends on certain characteristics of the structural element section. This paper studies the effects of adding fibers on the bending moment and shear force resistance depending on the geometry of the transversely unreinforced section, but also on the amount of longitudinal reinforcement. The results show a significant increase of the resisting moments for a high fiber content and reduced longitudinal reinforcement, but also a significant increase of the resisting shear force for a high fiber content, regardless of the amount of longitudinal reinforcement.*

Key words: *fiber reinforced concrete, bending moment, shear force, strength,*

Rezumat. *Diferite tipuri de fibre pot fi folosite pentru a genera o îmbunătățire a comportării mecanice a betonului armat dispers. Însă gradul în care aceasta se produce, depinde și de anumite caracteristici ale secțiunii elementului structural. În această lucrare sunt studiate efectele adăugării fibrelor în funcție de geometria secțiunii nearmate transversal, dar și de cantitatea de armătură longitudinală asupra rezistenței la moment încovoietor și forță tăietoare. Rezultatele arată o creștere semnificativă a momentelor capabile pentru un dozaj mare de fibre și armări longitudinale limitate, dar și o creștere semnificativă a forței tăietoare capabile pentru un dozaj mare, independent de cantitatea de armătură longitudinală.*

Cuvinte cheie: beton armat dispers, moment încovoietor, forță tăietoare, rezistență

1. Introduction

The beginning of fiber-reinforced concrete (FRC) as a genuine construction material can be traced back to the early 1960s. Fiber-reinforced concrete is a composite material combining a matrix (concrete) and reinforcement (fibers). The main role of the

fibers is to control cracking and absorb stress at any cracks that may develop. They give the concrete performance and properties related to their nature, shape, and mechanical characteristics. The range of uses for FRC has become extremely wide. FRC enriches the range of concrete construction solutions, thanks to the continuous development of a range of fibers with multiple properties.

Depending on their nature, fibers have specific geometric and mechanical characteristics and very different stress-strain behavior. The reinforcing capacity of a fiber depends in particular on its anchorage, tensile strength, and Young's modulus. Each has a particular influence on the mechanical behavior of concrete, which translates into specific, tailored applications. The choice of fiber type therefore depends on the field of application and the desired performance. Fibers are only useful if the concrete is subjected to tensile forces greater than its own strength. If cracks appear in the concrete, they allow the forces to be transmitted through the cracks.

Specific dimensioning methods (for structural applications: slabs, piles, etc.) and implementation techniques are now fully mastered for designing perfectly durable FRC structures.

Methods for optimizing their formulation have been specially developed. Fibers are highly compatible with the various components of concrete, including admixtures.

The structure of FRC and its mechanical characteristics also depend on how it is implemented (effects related to flow; preferential orientation of the fibers parallel to the direction of concrete flow and depending on the geometry of the structure). It is therefore necessary to know the implementation technique in order to develop the formulation and design of structures.

The expertise acquired through numerous research projects, FRC behavior tests, and multiple projects now makes it possible to characterize and specify FRC adapted to the performance requirements for each use. FRC can be formulated to be self-compacting and pumpable. Below, we will detail the behavior of this type of concrete and compare it to reinforced concrete. Finally, we will present a calculation example to illustrate the difference between this new type of concrete and traditional concrete.

2. Metal fibers and the composition of fiber-reinforced concrete

2.1. Metal fiber

This material is obtained by wire drawing and cutting rolled steel.

Whether glued, rounded, or with hooked ends, steel fiber comes in the form of steel wires with a diameter of one millimeter and a length of between 35 and 60 mm.

Micro-metal fibers, which are latest-generation materials, are also available on the market. This type of concrete offers optimal mechanical strength.

Metal fibers come in a variety of types and shapes and are highly compatible with concrete. Fig. 1 shows different metal fiber geometries.

- Straight metal fibers (a)
- Hooked metal fibers (b)
- Corrugated metal fibers (c)

Characterization of fiber reinforced concrete for classification based on strength and ductility

- Milled metal fibers (d)

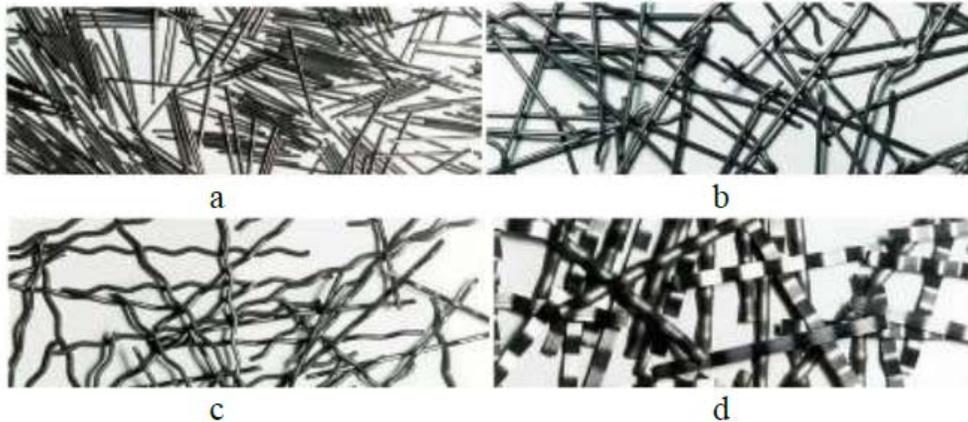


Fig. 1 Different types of fibers [1]

To use fibers, all the ingredients must be mixed for 2 minutes before adding water.

To improve concrete performance, fibers must:

- be flexible without being fragile;
- be relatively long and thin;
- have a large specific surface area;
- offer good deformation capacity;
- ensure good anchoring in the concrete;
- have good adhesion to the cement paste.

Thanks to their mechanical properties, fibers make it possible to better mobilize the intrinsic strength of concrete, produce large thin sections, and offer designers greater architectural freedom.

They give concrete many advantages:

- crack control;
- ease and speed of implementation;
- multidirectional and homogeneous reinforcement;
- partial or total replacement of traditional reinforcement

2.2. The composition of fiber-reinforced concrete

The composition of FRC is very similar to that of ordinary concrete. When designing the mix of fiber-reinforced concrete, the amount of fiber varies depending on the application and type of fiber. In addition, before adding fibers, it must be ensured that they are compatible with the other materials in the mix.

In order to obtain a fluid mix, which is not always easy in the presence of fibers, we must add an admixture. To achieve the correct workability, FRC often requires admixtures such as superplasticizers. These allow to obtain workable concrete, even in the presence of fibers.

The fracture pattern of FRC under tension shows the effect of the fibers: the fibers protrude from the crack surface, indicating that before fracture they connected the

opposite sides of the crack (Fig. 2). This is in fact the main way in which fibers can improve the mechanical behavior of concrete: the bridging effect across the crack, limiting its propagation and opening.



Fig. 2 Fracture pattern of FRC under tension [2]

3. Classification of FRC based on strength and ductility

The classification of FRC according to strength and ductility is carried out, in accordance with the new version of Eurocode 2[3], based on two characteristic values of residual strength obtained from the standard 3-point bending test on notched prisms [4,5]. These are $f_{R3,k}$, the residual strength value for a crack width $w = 2.5mm$, and $f_{R1,k}$, the residual strength value for a crack width $w = 0.5mm$. The value $f_{R3,k}$, rounded down to certain thresholds specified by the standard, gives the strength class, while the value of the ratio $f_{R3,k}/f_{R1,k}$, also rounded down to other thresholds, gives the ductility class. The strength class is expressed by the afore mentioned strength threshold in MPa, a number (1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 6, 7, 8), while the ductility class is expressed by a letter (a-e) associated with the ductility threshold (a for $f_{R3,k}/f_{R1,k} > 0.5$; b for $f_{R3,k}/f_{R1,k} > 0.7$; c for $f_{R3,k}/f_{R1,k} > 0.9$; d for $f_{R3,k}/f_{R1,k} > 1.1$ and e for $f_{R3,k}/f_{R1,k} > 0.5$).

4. Calculation of high-strength FRC at ultimate limit state

4.1. Bending calculation

In order to study the influence of certain parameters on the flexural strength and shear strength of a reinforced concrete element, it is first necessary to know the properties of the materials. However, in the specific case of FRC, it is not sufficient to consider only one compressive strength class. This is because the tensile strength class in a cracked state is related not only to the concrete, but also to the quality and quantity of the fibers. Consequently, interdependent concrete strength parameters occur. In this situation, we are led to consider properties corresponding to a real study of the literature (Table 1).

Table 1

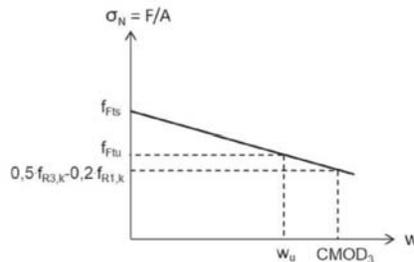
Strength and ductility of fiber reinforced concrete [6]		
Fiber amount	f_{ck}	Strength and ductility class
20 kg / m ³	75 MPa	1b
60 kg / m ³	75 MPa	5a

From this study, not only the strengths for two fiber contents will be used, but also the method for calculation of the resisting bending moment, the ultimate moment, noted $M_{u,d}$. This ultimate moment will be calculated not only for FRC, but also for non-fiber-reinforced concrete, in order to allow a comparison.

The calculation is based on the behavior laws of materials expressed by stress/strain relationships. Depending on the case, the stresses are tensile or compressive. The particularity of FRC consists in the fact that tensile strength is not negligible. Even if the fibers do not prevent the formation of cracks, they oppose their propagation and are subjected to tensile stress at the crack. Their contribution to the strength and stiffness of compressed concrete can be neglected, as it is not significant. Therefore, the behavior laws that need to be taken into account are those of compressed concrete, tension steel reinforcement (and compressed reinforcement, if any), and cracked fiber-reinforced concrete.

Those for compressed concrete and reinforcing steel are known from the strength classes. Behavior laws similar to those in Eurocode 2 [7] are used: parabolic/rectangular for compressed concrete and elastoplastic for steel. For FRC cracked under tension, a stress/crack opening relationship is used [3].

However, in order to determine the ultimate moment for an element at a given section, we do not a relationship between stress and crack opening, but a stress/strain relationship. The solution is to first calculate the relationship between stress and crack opening, and then convert it according to the characteristics of the element into a relationship between stress and strain, which is then taken into account. The tipe of constitutive relationship between stress and crack opening is that illustrated in the article by Buttignol et al (Fig. 3). It is therefore a constant stress f_{Ftu} for crack openings $w < w_u$. The value of the ultimate crack opening, w_u , is determined knowing that only values $w \leq 2.5 \text{ mm}$ are acceptable.


 Fig. 3 Stress (σ_N) / crack opening (w) curve for the analysis [6]

In this regard, the values of residual strength in service state, $f_{FTS,k}$, and ultimate limit state, $f_{Ftu3,k}$, are determined from the values $f_{R1,k}$ and $f_{R3,k}$ derived from the strength and ductility class (relations (1) and (2)).

$$f_{Fts,k} = 0,45 \cdot f_{r1,k} \quad (1)$$

$$f_{Ftu3,k} = 0,5 \cdot f_{r3,k} - 0,2 \cdot f_{r1,k} \quad (2)$$

After establishing the constitutive relationship of FRC in tension, the constitutive relationship of compressed concrete is determined. In this regard, the ultimate limit state uses the rectangular block of constant stress, defined based on the factors λ and η , in accordance with Eurocode 2 ((3) and (4)).

$$\lambda = 0,8 - \frac{(f_{ck} - 50)}{400} \quad (3)$$

$$\eta = 1 - \frac{(f_{ck} - 50)}{200} \quad (4)$$

For steel, a bilinear behavior law is used, with a horizontal yield point and no stress limit, in accordance with Eurocode 2.

As concerns the section geometry, a rectangular section is used, without compression reinforcement, with a width $b = 12.5 \text{ cm}$ and height $h = 25 \text{ cm}$, with two longitudinal bars with a diameter $\phi = 16 \text{ mm}$, strength class S500, and a cover $c = 20 \text{ mm}$.

At this point, it is possible to focus on the constitutive relationship of FRC, and more specifically on the conversion between the stress/crack opening relationship and the one used for calculation, expressed in stress/strain. The essential quantity for this method is the characteristic length, l_{cs} , which is essentially the distance between cracks. In this regard, it is necessary to know the effective height of the tension zone that acts as a tension tie, denoted $h_{c,ef}$. The calculation is performed according to the rules of Eurocode 2 [7], but with a change for the height of compressed concrete, x . For safety reasons, in order to consider the presence of steel fibers, the value corresponding to the ultimate limit state is considered, but neglecting the presence of fibers. The relationships for calculating x and $h_{c,ef}$ are (5) and (6). Consequently, the values of the area of the tie zone working in pure tension, $A_{c,ef}$, and the effective reinforcement ratio, $\rho_{s,f}$, are calculated from the relations (7) and (8).

$$x = \frac{A_s \cdot f_{yd}}{\lambda \cdot \eta \cdot f_{cd} \cdot b} \quad (5)$$

$$h_{c,ef} = \min \left(\left[2,5 \cdot (h - d) \frac{h - x}{3} \frac{h}{2} \right] \right) \quad (6)$$

$$A_{c,ef} = b \cdot h_{c,ef} \quad (7)$$

$$\rho_{s,f} = \frac{A_s}{A_{c,ef}} \quad (8)$$

Subsequently, the characteristic length l_{cs} is calculated according to the relationships in the article by Buttignol et al [6]. In order to do this, the average stress in the service state, $f_{Fts,m}$ and the average adhesion stress, τ_{bm} (relations (9) and (10)), are determined using the characteristic service stress, $f_{Fts,k}$ and the average tensile strength of the concrete, f_{ctm} . In this way, we determine the maximum transmission length between cracks, for average stresses, noted $l_{s,max}$ (11), and from this, the maximum distance between cracks, $s_{r,max}$ (12). The characteristic length, l_{cs} , becomes the maximum between the latter and y (the height of the tensioned zone) (13).

$$f_{Fts,m} = \frac{f_{Fts,k}}{0,7} \quad (9)$$

$$\tau_{bm} = 1,8 \cdot f_{ctm} \quad (10)$$

$$l_{s,max} = k \cdot c + \frac{1}{4} \cdot \frac{\Phi}{\rho_{s,ef}} \cdot \frac{f_{ctm} - f_{Fts,m}}{\tau_{bm}} \quad (11)$$

$$s_{r,max} = 1,5 \cdot l_{s,max} \quad (12)$$

$$l_{cs} = \min(s_{r,max} \quad y) \quad (13)$$

Thus, it is possible to determine the ultimate crack opening, w_u , as that corresponding to a strain $\varepsilon_{Fu} = 0.2\%$ (14). Subsequently, the ultimate characteristic stress, $f_{Ftu,k}$ (15), and the ultimate design stress, $f_{Ftu,d}$ (16), are determined using the partial safety coefficient, $\gamma_F = 1.5$.

$$w_u = \min([l_{cs} \cdot \varepsilon_{Fu} 2,5 \text{ mm}]) \quad (14)$$

$$f_{Ftu,k} = f_{Fts,k} - \frac{w_u}{2,5 \text{ mm}} \cdot f_{Ftu3,k} \quad (15)$$

$$f_{Ftu,d} = \frac{f_{Ftu,k}}{\gamma_F} \quad (16)$$

Then, from the stress-strain relationships of the three components of the section in the ultimate limit state (compressed concrete, steel and cracked concrete), it is possible to determine the resisting (ultimate) moment of the section. First, the height of the compressed concrete, x , is determined from the projection equation onto the section (17). Once this has been determined, it is possible to determine the compressive force in the concrete, R_{cd} (18), and its lever arm relative to the tensioned steel, z_c (19), as well as the tensile force in the cracked concrete, R_{fd} (20) and its lever arm relative to the tension steel, z_f (21). The value of the ultimate moment, M_{ud} , is determined from the moment equation with respect to the center of the tension reinforcement (22).

$$x = \frac{A_s \cdot \sigma_s + f_{Ftu,d} \cdot b \cdot h}{\eta \cdot 0,85 \cdot f_{cd} \cdot b + f_{Ftu,d} \cdot b} \quad (17)$$

$$R_{cd} = n \cdot 0,85 \cdot \frac{f_{ck}}{\gamma_c} \cdot b \cdot x \quad (18)$$

$$z_c = d - \frac{x}{2} \quad (19)$$

$$R_{fd} = f_{Ftu,d} \cdot b \cdot (h - x) \quad (20)$$

$$z_f = \frac{(h - x)}{2} - a_s \quad (21)$$

$$M_{ud} = R_{cd} \cdot z_c - R_{fd} \cdot z_f \quad (22)$$

For concrete without fibers, firstly, the height of the compressed concrete, x (23), is determined and then the value of the ultimate moment from the moment equation written with respect to the center of the tension reinforcement (24).

$$x = \frac{A_s \cdot f_{yd}}{b \cdot \eta \cdot f_{cd} \cdot 0,85} \quad (23)$$

$$M_{ud} = (b \cdot \eta \cdot x) \cdot f_{cd} \cdot 0,85 \cdot \left(d - \frac{x}{2}\right) \quad (24)$$

4.2. Shear force

The ultimate shear force of FRC reinforced longitudinally, but not transversely, was calculated according to relationship (25):

$$V_{Rd,f} = \left(C_{Rd,c} \cdot k \cdot \left(100 \cdot \rho_l \cdot \left(1 + 7.5 \cdot \frac{f_{Ftu,k}}{f_{ctk}} \right) \cdot f_{ck} \right)^{\frac{1}{3}} \right) \cdot b \cdot d \quad (25)$$

Where f_{ctk} is the tensile strength of concrete without fibers, while $f_{Ftu,k}$ is that of fiber concrete, obtained for a crack opening $w_u = 1.5 \text{ mm}$, and f_{ck} is the characteristic compressive strength of concrete.

The resisting shear force of reinforced concrete without fibers, not transversely reinforced, denoted $V_{Rd,c}$ is obtained using relationship (26):

$$V_{Rd,c} = \left(C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{\frac{1}{3}} \right) \cdot b \cdot d \quad (26)$$

5. Parametric study

In order to verify the effectiveness of adding fibers, a parametric study was carried out, focusing on two characteristics: the geometry of the section and the amount of longitudinal reinforcement. To this end, resisting (ultimate) moments and resisting shear forces were calculated for 3 section height/width ratios (noted h/b) and three reinforcement ratios (noted ρ_l). For these cases, the ratios between the resisting moments of FRC and non-FRC ($M_{ud,f}/M_{ud}$) and between the resisting shear forces of FRC and non-FRC ($V_{Rd,f}/V_{Rd,c}$) were calculated for the two fiber contents (20 kg/m^3 and 60 kg/m^3). These ratios were represented graphically by means of balls related to the unit reference (Fig. 4).

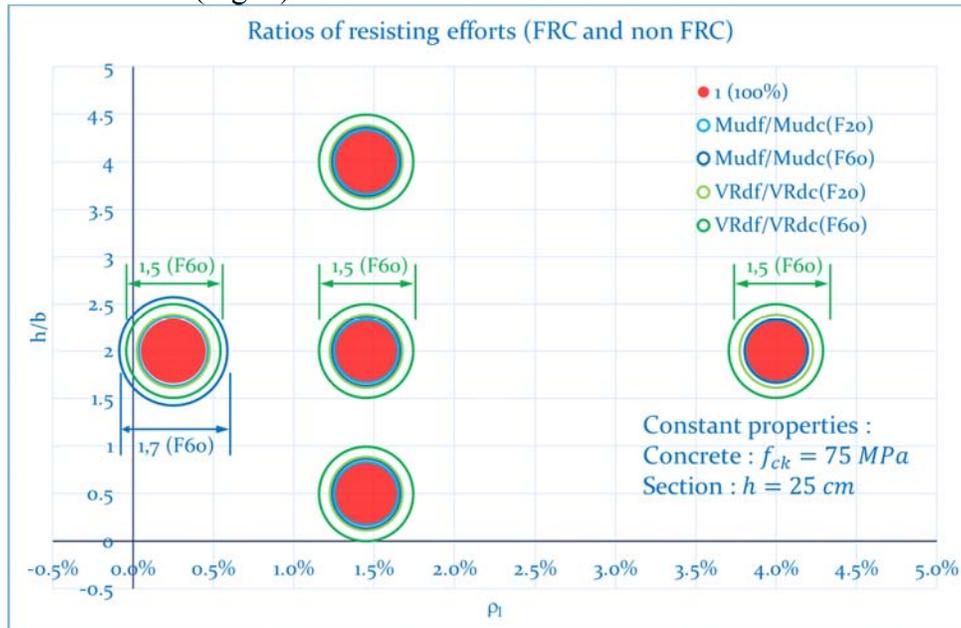


Fig. 4 Ratios between forces with and without fibers for different h/b ratios and longitudinal reinforcement ratios ρ_l

In this figure, we can see that the $M_{ud,f}/M_{ud}$ ratios vary between 1 and 1.11 for FRC with low fiber content ($20\text{ kg}/\text{m}^3$ shown in light blue) and between 1.02 and 1.7 for concrete with high fiber content ($60\text{ kg}/\text{m}^3$ shown in dark blue). The maximum ratios 1.11 and 1.7 were obtained for the lowest reinforcement ratio ($\rho_l = 0.25\%$) and are not influenced by the h/b ratio on the section (1.02 and 1.11 for the two contents respectively in the presence of $\rho_l = 1.45\%$). For the maximum reinforcement ratio, $\rho_l = 4\%$, the ratios $M_{ud,f}/M_{ud}$ are almost equal to 1 for both fiber contents, the strength increase brought about by the fibers being insignificant. These aspects demonstrate the high effectiveness of fiber addition, particularly in the region of low longitudinal reinforcement ratios and for higher fiber contents. The addition of fibers leads to an increase in ultimate moments, but this is significant particularly for low rates of longitudinal reinforcement, in the range of minimum reinforcement accepted by the standard. At low fiber contents, strength increases are fairly small, even for low longitudinal reinforcement ratios.

In terms of shear forces, the ratios $V_{Rd,f}/V_{Rd,c}$ range from 1.17 for FRC with low fiber content ($20\text{ kg}/\text{m}^3$ shown in light green) to 1.5 for FRC with high fiber content ($60\text{ kg}/\text{m}^3$ shown in dark green). These ratios are unaffected by either the section slenderness ratio, h/b , or the reinforcement ratio ρ_l . It can therefore be seen that the addition of fibers has a significant effect on shear strength, particularly for higher fiber contents. This observation is valid in the context where the reinforced concrete section considered is transversely unreinforced. Therefore, it would be possible to replace a certain amount of transverse reinforcement with a quantity of fibers that brings about the same increase of resistant shear force. Depending on the case, if the increase needed of resistant shear force is low, even a low fiber content could be sufficient, as its effect is not negligible.

6. Conclusion

In this study, the effect of fiber addition on bending moment and shear strength was investigated for a reinforced concrete element without transverse reinforcement, by varying the cross-section geometry and longitudinal reinforcement ratio.

Bending moment and shear force resistances were obtained in accordance with current standards and with certain adaptations for fiber-reinforced concrete, following procedures used in the literature.

The effect of adding fibers was found to be most significant in terms of increased bending moment, for low longitudinal reinforcement ratios but high fiber content. For low fiber content, the increase in bending moment resistance is quite small, even for low longitudinal reinforcement rates.

As far as shear strength is concerned, the effect is significant for high fiber content, and also not negligible for low contents either.

The practical utility of this study is to verify the possibility of replacing conventional reinforcement by means of steel rebar by adding fibers to the concrete to ensure a certain resisting effort, whether flexural or shear. The results show that this is

possible, particularly for high fiber contents. However, in terms of bending moment, fibers are only effective in the presence of low levels of longitudinal reinforcement.

7. References

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